Calculus 1
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Formulas for Exam 1

1. **Average and instantaneous rate of change.**
   (a) The average rate of change of $f(x)$ over $[a, b]$:
   $$\frac{f(b) - f(a)}{b - a}$$
   (b) The instantaneous rate of change of $f(x)$ at $x = a$: $\qquad f'(a)$.

2. **Definition of Derivative.** Derivative of $y = f(x)$ at $x = a$ is
   $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$
   In Leibniz notation, $\frac{dy}{dx}\Big|_{x=a} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$.

3. **Derivative from a table.** $f'(x_n) \approx$ average of $\frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$ and $\frac{f(x_{n+1}) - f(x_n)}{x_{n+1} - x_n}$.

4. **Power Rule.**
   $$y = x^n \quad \Rightarrow \quad y' = nx^{n-1}$$

5. **Tangent Line.** $y_0 = f(x_0)$, $m = f'(x_0)$
   $$y - y_0 = m(x - x_0)$$

6. **Sandwich Theorem.** If $g(x) \leq f(x) \leq h(x)$ and $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$ then $\lim_{x \to a} f(x) = L$.

7. **Applications.** The velocity $v(t) = s'(t) = \frac{ds}{dt}$ and the acceleration $a(t) = v'(t) = \frac{dv}{dt} = s''(t) = \frac{d^2 s}{dt^2}$.