

Formulas for the Final Exam

1. Derivatives.

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$

2. Integrals.

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$
$\int y \, dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b} b^x$	$\ln x $	$-\cos x$	$\sin x$

3. Rules of Differentiation.

(a) Product rule.

$$\text{If } y = f \cdot g, \text{ then } y' = f' \cdot g + g' \cdot f$$

(b) Quotient rule.

$$\text{If } y = \frac{f}{g}, \text{ then } y' = \frac{f' \cdot g - g' \cdot f}{g^2}$$

(c) Chain rule.

$$\text{If } y = f(g(x)), \text{ then } y' = f'(g(x)) \cdot g'(x)$$

4. **Derivative of the inverse function.** If $f(a) = b$ and f is invertible, then

$$(f^{-1})'(b) = \frac{1}{f'(a)}.$$

5. **Definition of Derivative.** Derivative of $y = f(x)$ at $x = a$ is

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}.$$

In Leibniz notation, $\left. \frac{dy}{dx} \right|_{x=a} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$.

6. **Average and instantaneous rate of change.**

(a) The average rate of change of $f(x)$ over $[a, b]$:

$$\frac{f(b) - f(a)}{b - a}$$

(b) The instantaneous rate of change of $f(x)$ at $x = a$: $f'(a)$.

7. **Derivative from a table.** $f'(x_n) \approx$ average of $\frac{f(x_n)-f(x_{n-1})}{x_n-x_{n-1}}$ and $\frac{f(x_{n+1})-f(x_n)}{x_{n+1}-x_n}$.

8. **Sandwich Theorem.** If $g(x) \leq f(x) \leq h(x)$ and $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L$ then $\lim_{x \rightarrow a} f(x) = L$.

9. **Linear Approximation.**

$$f(a + dx) \approx f(a) + f'(a)dx \quad \text{or} \quad f(x) \approx f(a) + f'(a)(x - a)$$

10. **Tangent Line.** $y_0 = f(x_0)$, $m = f'(x_0)$

$$y - y_0 = m(x - x_0)$$

11. **Velocity and Distance traveled.** The velocity $v(t) = s'(t) = \frac{ds}{dt}$ and $s(t) = \int v(t) dt$. The acceleration $a(t) = v'(t) = \frac{dv}{dt} = s''(t) = \frac{d^2s}{dt^2}$.

12. **Left and Right Sum.**

$$\text{Left sum} = \frac{b-a}{n} (f(x_0) + f(x_1) + \dots + f(x_{n-1}))$$

$$\text{Right sum} = \frac{b-a}{n} (f(x_1) + f(x_2) + \dots + f(x_n))$$

13. **Area between $f(x)$ and x -axis for $a < x < b$.**

- If $f(x) > 0$ for $a < x < b$, then area is $\int_a^b f(x) dx$

- If $f(x) < 0$ for $a < x < b$, then area is $-\int_a^b f(x) dx$

- If $f(x) < 0$ for $a < x < c$ and $f(x) > 0$ for $c < x < b$, then area is $-\int_a^c f(x) dx + \int_c^b f(x) dx$

Area between $f(x)$ and $g(x)$. if $f(x) > g(x)$ for $a < x < b$: $\int_a^b (f(x) - g(x)) dx$