Matlab Notes for Calculus 1

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Content

1. Basic Arithmetic

You can use +, -, *, \ and ^ to add, subtract, multiply, divide or exponentiate, respectively. For example if you enter:

>> 2^3 - 2*2
Matlab calculates the answer:
ans = 4

If you want to perform further calculations with the value of the answer, you can type **ans** rather than retyping the specific answer value. For example,
>> sqrt(ans)
ans = 2

To perform symbolic calculations in Matlab, use **syms** to declare the variables you plan to use. For example, suppose that you need factor x^2-3x+2. First you need
>> syms x (you are declaring that x is a variable)

Then you can use the command **factor**.
>> factor(x^2-3*x+2)
ans = (x-1)*(x-2)

Note that we entered **3*x** to represent $3x$ in the command above. **Entering * for multiplication is always necessary in Matlab.**

Besides **factor** command, you have **simplify** and **expand**.

2. Solving equations using "solve"

For solving equations, you can use the command **solve**. The command **solve** is always followed by parenthesis. After that, the equation you would like to solve should be entered in single quotes. Separated by a coma, the equation is followed by the variable for which you are solving the equation in (single) quotes. Thus, the command **solve** has the following form

solve('*equation***', '***variable for which you are solving***')**

For example, to solve the equation x^3-2x-4=0, you can use:
>> **solve('x^3-2*x-4=0')**
and get the following answer:
ans = [2] [-1+i] [-1-i]
Here *i* stands for the imaginary number $\sqrt{-1}$. This answer tells us that there is just one real solution, 2.

Matlab can give you both symbolic and numerical answer. For example, let us solve the equation 3x^2-8x+2=0.

>> **solve('3*x^2-8*x+2=0','x')**
ans = [4/3+1/3*10^(1/2)] [4/3-1/3*10^(1/2)]

If we want to get the answer in the decimal form with, say, three significant digits, we can use the command **vpa**.
>> vpa(ans, 3)
ans = [2.38] [0.28]

The command **vpa** has the general form
vpa(*expression you want to approximate***,** *number of significant digits***)**

You can solve an equation in two variables for one of them. For example the command
>> solve('y^2-5*x*y-y+6*x^2+x=2', 'y')
solves the given equation for values of *y* in terms of *x*. The answer is:
ans = [3*x+2] [2*x-1]

3. Representing a function

The following table gives an overview of how most commonly used functions or expressions are represented in Matlab.
To represent a function, use the command **inline**. Similarly to **solve**, this command is followed by parenthesis and has the following form:

function or symbol	**representation in MATLAB**
e^x	exp(x)
ln x	log(x)
log x	log(x)/log(10)
log. base a of x	log(x)/log(a)
sin x	sin(x)
cos x	cos(x)
arctan(x)	atan(x)
π	pi

inline('*function***', '***independent variable of the function***')**

Here is how to define the function x^2+3x-2:

>> f = inline('x^2+3*x-2', 'x')
f =
Inline function:
f(x) = x^2+3*x-2

After defining a function, we can evaluate it at a point. For example,

>> f(2) **ans = 8**

In some cases, we will need to define function f as a vector. Then we use:

>> f = inline(vectorize('x^2+3*x-2'), 'x')
f = Inline function: f(x) = x.^2+3.*x-2

In this case, we can evaluate a function at more than one point at the same time. For example, to evaluate the above function at 1, 3 and 5 we have:

>> f([1 3 5]) **ans = 2 16 38**

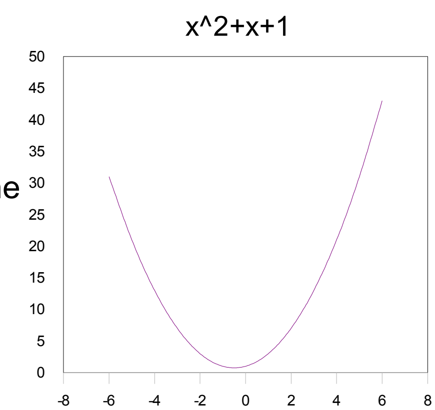
If a function is short, it might be **faster to evaluate a function at a point simply by typing the value of *x* directly for *x***. For example, you can evaluate sin(*x*) at *x*=2 as follows.

>> sin(2) **ans = .909297**

As when using the calculator, one must be careful when representing a function. For example

- $\frac{1}{x(x+6)}$ should be represented as **1/(x*(x+6))** not as **1/x*(x+6)** nor as **1/x(x+6),**
- $\frac{3}{x^2+5x+6}$ should be represented as **3/(x^2+5*x+6)** not as **3/x^2+5*x+6,**
- e^{5x^2} should be represented as **exp(5*x^2)** not as **e^(5*x^2)**, **exp*(5*x^2)**, **exp(5x^2)** nor as **exp^(5*x^2).**
- *ln*(x) should be represented as **log(x)**, not **ln(x).**
- $log_3(x^2)$ should be represented as **log(x^2)/log(3)** not as **log(x)/log(3)*x^2.**

4. Graphics



Let us start by declaring that *x* is a variable:

>> syms x

The simplest command in Matlab for graphing is **ezplot**. The command has the following form

ezplot(*function*)

For example, to graph the function x^2+x+1, you simply type

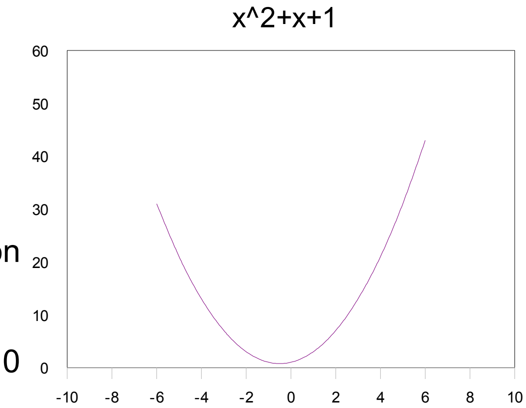
>> ezplot(x^2+x+1)

A new window will open and graph will be displayed. To copy the figure to a text file, go to **Edit** and choose **Copy Figure**. Then place cursor to the place in the word file where you want the figure to be pasted and choose **Edit** and **Paste**.

We can specify the different scale on *x* and *y* axis.
To do this, the command **axis** is used.
It has the following form

axis([x_{min}, x_{max}, y_{min}, y_{max}**])**

This command parallels the commands in menu WINDOW on the TI83 calculators.



For example, to see the above graph between x-values -10 and 10 and y-values 0 and 60, you can enter
>> axis([-10 10 0 60])

Note that the domain of function did not change by command axis. To see the graph on the entire domain (in this case [-10, 10]), add that domain after the function in the command ezplot:

ezplot(*function***, [**x_{min}, x_{max}**])**

In this case,
>> ezplot(x^2+x+1, [-10, 10])
will give you the desired graph.

For the alternative command for graphics, **plot,** you can find more details by typing **help**.

To graph multiple curves on the same plot, you can also use the **ezplot** command.

To graph multiple curves on the same window, you can use the **ezplot** command in combination with hold on and hold off on the following way:

ezplot(*1st function***)**
hold on
ezplot(*2nd function***)**
ezplot(*3rd function)*
...
ezplot(*n-th function***)**
hold off

For example to graph the functions sin(*x*) and $e^{-x^{\wedge}2}$, you can use: **>> ezplot(sin(x))**
>> hold on **>> ezplot(exp(-x^2))** **>> hold off**

5. Solving equations using “fzero”

In some cases, the command **solve** may fail to produce all the solutions of an equation. In those cases, you can try to find solutions using **fzero** (short for "find zero") command. In order to use the command, first you need to write equation in the form

f(x)=0.

Thus, <u>put all the terms of th equations on one side</u> leaving just zero on the other. To find a solution near the *x*-value x=*a*, you can use

fzero('*left side of the equation***',** *a***)**

The command **fzero**, similarly as **solve** is always followed by expression in parenthesis. The equation should be in single quotes.

If it is not clear what a convenient *x*-value *a* should be, you may want to graph the function on the left side of the equation first, check where it intersects the *x*-axis. Alternatively, you can graph left and right side of the equation that is not in *f(x)=0* form and see where the two functions intersect. Then decide which x-value you should use.

Example. To solve the equation $e^{x^{\wedge}2}-2=x+4$, we can first graph the functions on the left and right side of the equation using

syms x   ezplot(exp(x^2)-2)   hold on   ezplot(x+4)   hold off

From the graph, we can see that the two functions intersect at a value near -1 and at a value near 1. To use **fzero**, we need to represent the equation in the form $e^{x^{\wedge}2}-2-(x+4)=0$ (or simplified form $e^{x^{\wedge}2}-x-6=0$). Then, we can find the positive solution by using **fzero** to find a zero near 1 and then to find the negative solution near -1, for example. Thus, both solutions can be obtained by:

>> fzero('exp(x^2)-2-(x+4)', 1)   ans = 1.415
>> fzero('exp(x^2)-2-(x+4)', -1)   ans = -1.248

Note also that the command **solve('exp(x^2)-2=x+4', 'x')** returns just the positive solution. Thus, knowing how to use **fzero** command may be really useful in some cases.

6. Limits

You can use **limit** to compute limits, left and right limits as well as infinite limits. For example, to evaluate the limit when $x \to 2$ of the function $\frac{x^2-4}{x-2}$, we have:

>> syms x
>> limit((x^2-4)/(x-2), x, 2)   ans = 4

You can also evaluate left and right limits. For example:
>> limit(abs(x)/x, x, 0, 'left')   ans = -1
>> limit(abs(x)/x, x, 0, 'right')   ans = 1

Limits at infinity:
>> limit(exp(-x^2-5)+3, x, Inf)   ans = 3

7. Differentiation

Start by declaring x for a variable. The command for differentiation is **diff**. It has the following form

diff(*function***)**

For example,
>> syms x
>> diff(x^3-2*x+5) **ans = 3*x^2-2**

To get n-th derivative use

diff(*function, n***)**

For example, to get the second derivative of x^3-2x+5, use: **>> diff(x^3-2*x+5, 2)**
ans = 6*x
Similarly, the 23rd derivative of sin(x) is obtained as follows. **>> diff(sin(x), 23)**
ans =-cos(x)

To evaluate derivative at a point, we need to represent the derivative as a new function. For example, to find the slope of a tangent line to x^2+3x-2 at point 2, we need to find the derivative and to evaluate it at $x=2$.
>> diff(x^2+3*x-2) (first we find the derivative) **ans = 2*x+3**
>> f = inline('2*x+3', 'x') (then we representative the derivative as a function)
f = Inline function: f(x) = 2*x+3
>> f(2) (and, finally, we evaluate the derivative at 2) **ans = 7**

8. Optimization

Recall the steps needed in order to find minimum or maximum values of a given function (using second derivative test)

- Find first derivative
- Solve it for zeros. The x-values you obtain are called critical
- Find second derivative
- Plug critical points in second derivative. If your answer is negative, the function has a maximum value at a critical point used. If your answer is positive, the function has a minimum value at a critical point used.
- Plug critical points in your function. The y-values you obtain are your maximum or minimum values.

In MATLAB, start with **syms x**.

1. Finding derivative: **diff(***function***)**
2. Finding critical points: **solve('***copy-paste the answer from step 1***=0', 'x')**
3. Finding second derivative: **diff(***function, 2***)**
4. Evaluating second derivative at critical points: **g=inline(***second derivative***, 'x')** followed by **g(***critical value***)**

5. Evaluating function at critical points: **f=inline(*function*, 'x')** followed by **f(*critical value*)**

For example, to find extreme values of x^3-2x+5, start by finding first derivative:
>> diff(x^3-2*x+5) **ans = 3*x^2-2**
Then find critical point(s):
>> solve('3*x^2-2=0', 'x') **ans = [6^(1/2)/3] [-6^(1/2)/3]**
or, using **vpa(ans, 3)** **ans = [.816] [-.816]**

Find second derivative **>> diff(x^3-2*x+5, 2)** **ans = 6*x**
Evaluate this at critical points. **>> g=inline('6*x', 'x')** **g(x)= 6*x**
>>g(.816) **ans = 4.896**
Positive answer means that the function has minimum at x=.816
>> g(-.816) **ans = -4.896**
Negative answer means that the function has maximum at x=.816

Finding y-values of maximum and minimum:
>> f=inline('x^3-2*x+5', 'x') **f(x)= x^3-2*x+5**
>>f(.816) **ans = 3.911** This is the local minimum value.
>>f(.816) **ans = 6.088** This is the local maximum value.

9. Integration

We can use Matlab for computing both definite and indefinite integrals using the command **int**. For the indefinite integrals, start with **syms x** followed by the command

int(*function*)

For example, the command
>> int(x^2)
evaluates the integral $\int x^2 dx$ and gives us the answer **ans = 1/3*x^3**

For definitive integrals, the command is

int(*function, lower bound, upper bound*)

For example,
>> int(x^2, 0, 1)
evaluates the integral $\int_0^1 x^2 dx$ The answer is **ans = 1/3**

Matlab can evaluate the definitive integrals of the functions that do not have elementary primitive functions. Recall that the integrals $\int \frac{\sin x}{x} dx, \ \int \frac{e^x}{x} dx, \ \int e^{x^2} dx$
can not be represented via elementary functions. Suppose that we need to find the integral of $\frac{\sin x}{x}$ from 1 to 3. The command >> **int(sin(x)/x, 1, 3)**
doesn't gives us a numerical value. We have just: **ans =** **sinint(3)-sinint(1)**
Using the command **vpa,** we obtain the answer in numerical form. For example,
>> vpa(ans, 4) gives us **ans = 0.9026**

10. Practice problems

1. Factor $x^3+3x^2y+3xy^2+y^3$. 2. Simplify $\frac{x^3-8}{x-2}$.

3. Evaluate the following expressions. (a) sin(π/6) (b) $\frac{\sqrt{5}+3}{\sqrt{3}-1}$ (c) $\log_2(5)$

4. Solve the following equations and express the answers as decimal numbers.
   (a) $x^3-2x+5=0$ (b) $\log_2(x^2-9)=4$.

5. Let f(x)= $\frac{x^3+x+1}{x}$ (a) Represent *f(x)* as a function in Matlab and evaluate it at 3 and -2.
(b) Find *x*-value(s) that corresponds to *y*-value y=2. (c) Graph *f(x)* on domain [-4 4].

6. Graph ln(*x*+1) and $1-x^2$ on the same plot for *x* in [-2 6] and *y* in [-4 4].

7. Find the limits of the following functions at indicated values.
(a) f(x)= $\frac{x^{12}-1}{x^3-1}$, $x \to 1$ (b) f(x)= $3+e^{-2x}$, $x \to \infty$ (c) f(x)= $\frac{6x^3-4x+5}{2x^3-1}$, $x \to \infty$

8. Let f(x)= $\frac{x^3+x+1}{x}$ Find the first derivative of *f(x)* and evaluate it at *x*=1.

9. Let f(x)=$e^{3x\wedge 2+1}$. (a) Find the first derivative of *f(x)*. (b) Find the slope of the tangent line to *f(x)* at x=1. (c) Find the critical points of f(x).

10. Find the 12th derivative of the function $(\frac{x}{2}+1)^{65}$.

11. Find the extreme values of (a) x^3-4x+8 (b) xe^{-3x}

12. Evaluate the following integrals. (a) $\int xe^{-3x}dx$ (b) $\int_0^1 xe^{-3x}dx$.

<u>Solutions</u>.

1. **syms x y** followed by **factor(x^3+3*x^2*y+3*x*y^2+y^3)** gives you **ans=(x+y)^3**
2. **syms x** followed by **simplify((x^3-8)/(x-2))** gives you **ans=x^2+2x+4**
3. (a) **sin(pi/6) ans=.5** (b) **(sqrt(5)+3)/(sqrt(3)-1) ans=7.152** (c) **log(5)/log(2) ans=2.3219**.
4. (a) **solve('x^3-2*x+5=0', 'x') ans= -2.09.** (b) **solve('log(x^2-9)/log(2)=4','x'). ans= 5, -5.**
5. (a) **>> f=inline('(x^3+x+1)/x', 'x')**, **>> f(3) ans= 10.333, >>f(-2) ans=4.5.**

(b) The problem is asking you to solve equation $\frac{x^3+x+1}{x}$ =2. Using solve command, **solve('(x^3+x+1)/x=2','x')**. you get **ans=-1.3247** (c) **ezplot((x^3+x+1)/x, [-4,4]).**

6. **hold on ezplot(log(x+1)) ezplot(1-x^2) hold off axis([-2 6 -4 4])**
7. (a) **syms x limit((x^12-1)/(x^3-1), x, 1) ans=4**
(b) **limit(3+exp(-2*x), x, Inf) ans=3** (c) **limit((6*x^3-4*x+5)/(2*x^3-1), x, Inf) ans=3**
8. (a) **syms x diff((x^3+x+1)/x) ans = 2*x-1/x^2** or **(2*x^3-1)/x^2.**
   (b) Inline the derivative: **g=inline('2*x-1/x^2','x')**. Then **g(1)** gives you **ans=1**.
9. (a) **diff(exp(3*x^2+1)) ans=6*x*exp(3*x^2+1)**
(b) Represent the derivative as function: **g=inline('6*x*exp(3*x^2+1)','x').** Then evaluate **g(1)**. Get 6*exp(4). To see the answer as a decimal number (say to five nonzero digits) use vpa(ans, 5). Get 327.58.
   (c) **solve('6*x*exp(3*x^2+1)=0','x') ans=0**
10. **diff((x/2+1)^65, 12)**
11. (a) max (-1.15, 11.079), min (1.15, 4.92). (b) max (.333, .1226), no min.
12. (a) **syms x int(x*exp(-3*x)) ans=-1/3*x*exp(-3*x)-1/9*exp(-3*x)**
    (b) **int(x*exp(-3*x), 0,1) ans=-4/9*exp(-3)+1/9 vpa(ans, 4) ans=.08898**