Calculus 2
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Formulas for Exam 1

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

For derivatives and integrals of $\cos^{-1} x$, $\cot^{-1} x$, and $\csc^{-1} x$ add minus sign to formulas for $\sin^{-1} x$, $\tan^{-1} x$, and $\sec^{-1} x$ respectively.

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \lvert x \rvert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Rules of Differentiation**

a) Product rule:

$$\text{If} \quad y = f \cdot g, \quad \text{then} \quad y' = f' \cdot g + g' \cdot f$$

b) Quotient rule:

$$\text{If} \quad y = \frac{f}{g}, \quad \text{then} \quad y' = \frac{f' \cdot g - g' \cdot f}{g^2}$$

c) Chain rule:

$$\text{If} \quad y = f(g(x)), \quad \text{then} \quad y' = f'(g(x)) \cdot g'(x)$$

4. **Area between $f(x)$ and x-axis for $a < x < b$:**

- If $f(x) > 0$ for $a < x < b$, then area is $\int_a^b f(x)\, dx$
- If $f(x) < 0$ for $a < x < b$, then area is $-\int_a^b f(x)\, dx$
- If $f(x) < 0$ for $a < x < c$ and $f(x) > 0$ for $c < x < b$, then area is $-\int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

Area between $f(x)$ and $g(x)$ for $a < x < b$:

- If $f(x) > g(x)$ for $a < x < b$, then area is $\int_a^b (f(x) - g(x))\, dx$
- If $f(x) < g(x)$ for $a < x < c$ and $f(x) > g(x)$ for $c < x < b$, then area is $\int_a^c (g(x) - f(x))\, dx + \int_c^b (f(x) - g(x))\, dx$

5. **The volume of the solid of revolution.**

- Revolving $f(x)$ on $[a, b]$, axis of revolution x-axis. Volume

$$V = \int_a^b \pi (f(x))^2 \; dx$$

- Revolving region between $f(x)$ and $g(x)$, $|f(x)| > |g(x)|$ on $[a, b]$, axis of revolution x-axis. Volume

$$V = \int_a^b \pi((f(x))^2 - (g(x))^2) \ dx$$

- Revolving $f(x)$ on $[a, b]$, axis of revolution y-axis. Volume

$$V = \int_a^b 2\pi x \ f(x) \ dx$$

- Revolving region between $f(x)$and $g(x)$, $f(x) > g(x)$ on $[a, b]$, axis of revolution y-axis. Volume

$$V = \int_a^b 2\pi x \ (f(x) - g(x)) \ dx$$

6. **Approximate integration.**

$$\text{Left sum} = \frac{b-a}{n} \ (f(x_0) + f(x_1) + \ldots + f(x_{n-1}))$$

$$\text{Right sum} = \frac{b-a}{n} \ (f(x_1) + f(x_2) + \ldots + f(x_n))$$

7. **Properties of logarithmic function.**

- $\log_a(x \cdot y) = \log_a x + \log_a y$
- $\log_a(x/y) = \log_a x - \log_a y$
- $\log_a(x^r) = r \log_a x$
- $\log_a x = \ln x / \ln a$

8. **Average value and average rate of change.**

- Average value:

$$f_{\text{ave}} = f(c) = \frac{1}{b-a} \int_a^b f(x) \ dx.$$

- Average rate of change: $f'_{\text{ave}} = \frac{1}{b-a} \int_a^b f'(x)dx = \frac{f(b)-f(a)}{b-a}$.

9. **Finding the second solution of some basic trigonometric equations.**

$$\begin{aligned}
&\sin x = a \quad \Rightarrow \quad x_1 = \sin^{-1}(a) \quad \text{and} \quad x_2 = \pi - \sin^{-1}(a) \\
&\cos x = a \quad \Rightarrow \quad x_1 = \cos^{-1}(a) \quad \text{and} \quad x_2 = -\cos^{-1}(a) \\
&\tan x = a \quad \Rightarrow \quad x_1 = \tan^{-1}(a) \quad \text{and} \quad x_2 = \pi + \tan^{-1}(a)
\end{aligned}$$

10. **L'Hôpital's Rule**

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

11. **Work** $= \int_a^b \text{force} \ dx$. For the spring use the Hook's Law: force $= kx$, work $= \int_a^b kx \ dx$.

12. **Miscellaneous**. Point-slope equation of a line. $y - y_1 = m(x - x_1)$