

Formulas for Exam 2

1. Derivatives.

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$

2. Integrals.

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b} b^x$	$\ln x $	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. Rules of Differentiation

- Product rule: If $y = f \cdot g$, then $y' = f' \cdot g + g' \cdot f$
- Quotient rule: If $y = \frac{f}{g}$, then $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$
- Chain rule: If $y = f(g(x))$, then $y' = f'(g(x)) \cdot g'(x)$

4. Integration by parts.

$$\int u dv = uv - \int v du$$

- Integrals with polynomial and e^x . Try $u =$ polynomial.
- Integrals with polynomial and $\sin x$ (or $\cos x$). Try $u =$ polynomial.
- $\int e^{ax} \sin bx dx$ or $\int e^{ax} \cos bx dx$. You can start with $u = e^{ax}$. You will need to do integration by parts twice.
- Integrals with log function. Try $u =$ log function.
- Integrals with inverse trig. functions. Try $u =$ inverse trig. function.

5. L'Hôpital's Rule

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

6. Approximate integration.

$$\text{Left sum} = \frac{b-a}{n} (f(x_0) + f(x_1) + \dots + f(x_{n-1}))$$

$$\text{Right sum} = \frac{b-a}{n} (f(x_1) + f(x_2) + \dots + f(x_n))$$

$$\begin{aligned} \text{Trapezoidal sum} &= \frac{b-a}{2n} (f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)) \\ &= \frac{1}{2}(\text{Left sum} + \text{Right sum}) \end{aligned}$$

$$\begin{aligned} \text{Simpson's sum} &= \frac{b-a}{3n} (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots \\ &\dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)) \end{aligned}$$

7. The arc length.

$$L = \int_a^b \sqrt{1 + (y')^2} \, dx.$$

8. The surface area.

- Surface area. Revolving $y = y(x)$ on $[a, b]$, axis of revolution x -axis.

$$S = \int_a^b 2\pi y \sqrt{1 + (y')^2} \, dx$$

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9. Applications.

- If A is the area of the region bounded by $y = f(x)$ from above and $y = g(x)$ from below for $a \leq x \leq b$, then the coordinates (\bar{x}, \bar{y}) of the center of mass are

$$\bar{x} = \frac{1}{A} \int_a^b x(f(x) - g(x)) \, dx \quad \bar{y} = \frac{1}{A} \int_a^b \frac{1}{2}((f(x))^2 - (g(x))^2) \, dx$$

- If $c(t)$ is the concentration of the dye.

$$\text{Flux, cardiac output} = \frac{\text{amount of the dye } A}{\int_0^T c(t) dt}.$$

10. Properties of logarithmic function.

- $\log_a(x \cdot y) = \log_a x + \log_a y$
- $\log_a(x/y) = \log_a x - \log_a y$
- $\log_a(x^r) = r \log_a x$
- $\log_a x = \ln x / \ln a$

11. Trigonometric identities.

- $\sin^2 x + \cos^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin x \cos x = \frac{1}{2} \sin 2x$