

Formulas for Exam 3 and the Final Exam

1. Derivatives.

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. Integrals.

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b} b^x$	$\ln x $	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. Rules of Differentiation

a) Product rule: If $y = f \cdot g$, then $y' = f' \cdot g + g' \cdot f$

b) Quotient rule: If $y = \frac{f}{g}$, then $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$

c) Chain rule: If $y = f(g(x))$, then $y' = f'(g(x)) \cdot g'(x)$

4. Integration by parts. $\int u dv = uv - \int v du$

5. Area between $f(x) > 0$ and x -axis for $a < x < b$: $\int_a^b f(x) dx$

Area between $f(x)$ and $g(x)$, $f(x) > g(x)$, for $a < x < b$: $\int_a^b (f(x) - g(x)) dx$

6. The volume of the solid of revolution.

- axis of revolution x -axis: $V = \int_a^b \pi (f(x))^2 dx$ or $V = \int_a^b \pi ((f(x))^2 - (g(x))^2) dx$

- axis of revolution y -axis: $V = \int_a^b 2\pi x f(x) dx$ or $V = \int_a^b 2\pi x (f(x) - g(x)) dx$

7. The arc length. $L = \int_a^b \sqrt{1 + (y')^2} dx$.

8. The surface area.

- axis of revolution x -axis, revolving $y = y(x)$ on $[a, b] \Rightarrow S = \int_a^b 2\pi y \sqrt{1 + (y')^2} dx$

- axis of revolution y -axis, revolving $y = y(x)$ on $[a, b] \Rightarrow S = \int_a^b 2\pi x \sqrt{1 + (y')^2} dx$

9. Approximate integration.

- Left sum = $\frac{b-a}{n} (f(x_0) + f(x_1) + \dots + f(x_{n-1}))$

- Right sum = $\frac{b-a}{n} (f(x_1) + f(x_2) + \dots + f(x_n))$

- Trapezoidal sum = $\frac{b-a}{2n} (f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)) = \frac{1}{2}(\text{Left} + \text{Right})$

- Simpson's sum = $\frac{b-a}{3n} (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$

10. Properties of logarithmic function.

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

$$\log_a(x/y) = \log_a x - \log_a y$$

$$\log_a(x^r) = r \log_a x$$

$$\log_a x = \ln x / \ln a$$

11. Properties of trigonometric functions.

$$\sin^2 x + \cos^2 x = 1$$

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin x = a \Rightarrow x_1 = \sin^{-1}(a) \quad \text{and} \quad x_2 = \pi - \sin^{-1}(a)$$

$$\cos x = a \Rightarrow x_1 = \cos^{-1}(a) \quad \text{and} \quad x_2 = -\cos^{-1}(a)$$

12. Applications.

- Work = \int_a^b force dx . For the spring: force $F = kx$, work = $\int_a^b kx dx$.

- Average value: $f_{\text{ave}} = f(c) = \frac{1}{b-a} \int_a^b f(x) dx$.

- Point-slope equation of a line. $y - y_1 = m(x - x_1)$

- Coordinates (\bar{x}, \bar{y}) of the center of mass: $\bar{x} = \frac{1}{A} \int_a^b x(f(x) - g(x)) dx$

$$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2}((f(x))^2 - (g(x))^2) dx$$

- If $c(t)$ is the concentration of the dye. Flux, cardiac output = $\frac{\text{amount of the dye } A}{\int_0^T c(t) dt}$.

13. **Linear Differential Equation:** $y' + P(x)y = Q(x)$. Integrating factor: $I(x) = e^{\int P(x) dx}$. After multiplying with $I(x)$, left side of the equation is equal to derivative of $I(x) \cdot y$.

14. **Parametric Curve** $x = x(t)$ and $y = y(t)$, $t_1 \leq t \leq t_2$.

- Derivative: $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$. Second derivative: $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{y'(t)}{x'(t)} \right)}{x'(t)}$.

- Area: $A = \int_a^b y dx = \pm \int_{t_1}^{t_2} y(t)x'(t) dt$

- Arc length: $L = \int_{t_1}^{t_2} \sqrt{(x')^2 + (y')^2} dt$

- Surface area around x -axis: $S = \int_{t_1}^{t_2} 2\pi y \sqrt{(x')^2 + (y')^2} dt$

- Surface area around y -axis: $S = \int_{t_1}^{t_2} 2\pi x \sqrt{(x')^2 + (y')^2} dt$

15. Polar Coordinates.

- Relation between Cartesian and polar coordinates:

$$x = r \cos \theta, \quad y = r \sin \theta \quad \text{and} \quad r = \sqrt{x^2 + y^2}, \quad \tan \theta = \frac{y}{x}.$$

- If $r = r(\theta) \Rightarrow x = r(\theta) \cos \theta$ and $y = r(\theta) \sin \theta$ then the derivative of r is $\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)}$.

- Area: $A = \int_{\alpha}^{\beta} \frac{1}{2}(r(\theta))^2 d\theta$

- Area between $r = f(\theta)$ and $r = g(\theta)$: $A = \int_{\alpha}^{\beta} \frac{1}{2}((f(\theta))^2 - (g(\theta))^2) d\theta$

- Arc length: $L = \int_{\alpha}^{\beta} \sqrt{(r(\theta))^2 + (r'(\theta))^2} d\theta$.

16. Taylor Polynomial at $x = a$ of order n .

$$f(x) \approx \sum_{i=0}^n \frac{f^{(i)}(a)}{i!} (x-a)^i = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \frac{f'''(a)}{3!} (x-a)^3 + \dots + \frac{f^{(n)}(a)}{n!} (x-a)^n$$