

## Review for Exam 2

a) **Integration.** Evaluate the integrals:

1.

$$\int \frac{x - 9}{(x + 5)(x - 2)} dx$$

2.

$$\int 2x \sin 3x dx$$

3.

$$\int \frac{\ln(2x - 1)}{(2x - 1)^2} dx$$

4.

$$\int_1^{\infty} \frac{1}{x^3} dx$$

5.

$$\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx$$

6.

$$\int \frac{2x^2 + x + 1}{x^3 + x} dx$$

7.

$$\int_0^{\infty} x e^{-2x} dx$$

8.

$$\int \sin^{10} x \cos x dx$$

9.

$$\int \sin^3 x \cos^2 x dx$$

10.

$$\int 3 \sin^{-1} 2x dx$$

11.

$$\int e^x \sin x dx$$

12.

$$\int \frac{x^2 + 1}{x^2 - x} dx$$

13.

$$\int x^2 e^x dx$$

14.

$$\int \sin^2 x dx$$

15.

$$\int \sin^5 x dx$$

16.

$$\int \frac{x+1}{x^3-2x^2+x} dx$$

17.

$$\int \frac{2x^2-4x}{(x-1)^2(x^2+1)} dx$$

b) **Unbounded area. Improper integrals.** Sketch the following region and find its area if the area is finite.

1.  $x \geq 3$ ,  $0 \leq y \leq \frac{1}{(x-2)^2}$
2.  $x \geq 0$ ,  $0 \leq y \leq \frac{1}{(x+2)(x+3)}$
3.  $x \geq 1$ ,  $0 \leq y \leq \frac{\ln x}{x^2}$

c) **The arc length.**

1. Find the length of the curve  $y = x^{3/2}$ ,  $1 \leq x \leq 4$
2. Find the length of the curve  $y = \sqrt{1-x^2}$ ,  $-1 \leq x \leq 1$
3. Use the Left-Right sum calculator program with  $n = 100$  subintervals to approximate the length of the curve  $y = e^x$ , for  $0 \leq x \leq 1$ .
4. Use the Left-Right sum calculator program to approximate the length of the curve  $y = \sin x$ , for  $0 \leq x \leq \pi$  to five digits.

d) **The surface area.** Find the area of the surface obtained by rotating the given curve about the specified line.

1.  $y = x^3$ ,  $0 \leq x \leq 2$  about the  $x$ -axis.
2.  $y = \sqrt{x}$ ,  $4 \leq x \leq 9$  about the  $x$ -axis.
3.  $y = x^2$ ,  $1 \leq x \leq 2$  about the  $y$ -axis.
4. Use the Left-Right sum calculator program to approximate the surface area obtained by rotating the curve  $y = \sin x$ , for  $0 \leq x \leq \pi$  about  $x$ -axis to four digits.
5. Use the Left-Right sum calculator program with 100 subintervals to find the Right sum which approximates the surface area of the surface obtained by rotating  $y = \ln(x^3 + 1)$   $0 \leq x \leq 1$ , about  $y$ -axis.

e) **Approximate integration.**

1. A CAT scan produces equally spaced cross-sectional views of a human organ that provide information about the organ otherwise obtained only by surgery. Suppose that a CAT scan of a human liver shows cross-sections spaced 2 cm apart. The liver is 12cm long and the cross-sectional areas, in square centimeters are 0, 58, 94, 106, 117, 63, 0. Use the Trapezoidal Sum to approximate the volume of the liver.
2. A chemical reaction produces a compound X with a rate of 23, 19, 12, 11, 9, 5, 2 liters per second at time intervals spaced by 1 second. Approximate the total volume of the compound X produced in the 6 seconds for which the rate is given using the Simpson's Sum.
3. A 5 mg bolus of dye is injected into the right atrium. The concentration of dye (mg/l) is measured in the aorta at one-second intervals as shown.

$t$	0	1	2	3	4	5	6	7	8	9	10
$c(t)$	0	.4	2.8	6.5	9.8	8.9	6.1	4	2.3	1.1	0

Use Simpson's Rule to estimate the cardiac output.

4. A 10 mg bolus of dye is injected into the right atrium. The concentration of dye (mg/l) is measured in the aorta at one-second intervals as shown.

$t$	0	1	2	3	4	5	6	7	8
$c(t)$	0	0.5	2.4	6.1	8.3	6.3	4.1	1.6	0.4

Use Trapezoidal Rule to estimate the cardiac output.

f) **Center of Mass.**

1. Find the center of mass of the region bounded by  $y = \sin x$ ,  $y = \cos x$ ,  $x = 0$ ,  $x = \pi/4$ .
2. Find the center of mass of the region bounded by  $y = \sin 2x$ ,  $y = 0$ ,  $x = 0$ ,  $x = \pi/2$ .

## Review 2 – Solutions

The class handouts contain more detailed solutions of these problems.

### a) Integration

1.  $2 \ln |x + 5| - \ln |x - 2| + c$
2.  $\frac{-2}{3}x \cos 3x + \frac{2}{9} \sin 3x + c$
3.  $\frac{-\ln(2x-1)}{2(2x-1)} - \frac{1}{2(2x-1)} + c$
4. Convergent,  $\frac{1}{2}$
5.  $\frac{1}{x} + 2 \ln |x| + 3 \ln |2 + x| + c$
6.  $\ln x + \frac{1}{2} \ln(1 + x^2) + \tan^{-1} x + c$
7. Convergent,  $\frac{1}{4}$
8.  $\frac{1}{11} \sin^{11} x + c$
9.  $\frac{-1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + c$
10.  $3x \sin^{-1} 2x + \frac{3}{2} \sqrt{1 - 4x^2} + c$
11.  $\frac{-1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + c$
12.  $x - \ln |x| + 2 \ln |x - 1| + c$
13.  $x^2 e^x - 2x e^x + 2e^x + c$
14.  $\frac{1}{2}x - \frac{1}{4} \sin 2x + c$
15.  $-\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + c.$
16.  $\ln x - \frac{2}{x-1} - \ln |x - 1| + c$
17.  $\ln |x - 1| + \frac{1}{x-1} - \frac{1}{2} \ln(1 + x^2) + 2 \tan^{-1} x + c$

### b) Unbounded area. Improper integrals.

1.  $A = \int_3^\infty \frac{1}{(x-2)^2} dx$ . This integral is convergent and equal to 1 so  $A = 1$ .
2.  $A = \int_0^\infty \frac{1}{(x+2)(x+3)} dx$  This integral is convergent and equal to  $A = \ln \frac{3}{2} \approx 0.405$ .
3.  $A = \int_1^\infty \frac{\ln x}{x^2} dx$ . This integral is convergent and equal to 1 so  $A = 1$ .

c) The arc length. 1. 7.6337      2.  $\pi$       3. 2.00      4. 3.8202

d) The surface area. 1. 203.0436      2. 81.1418      3. 30.8465      4. 14.42      5. 4.54

e) Approximate integration. 1. 867 cm<sup>3</sup>.      2. 69 liters      3. .12 liters per second      4. .34 liters per second

f) Center of Mass. 1.  $A = .414, (\bar{x}, \bar{y}) = (.267, .6035)$       2.  $A = 1, (\bar{x}, \bar{y}) = (\frac{\pi}{4}, \frac{\pi}{8})$