Calculus 2
Lia Vas

Review for Exam 2

a) **Integration.** Evaluate the integrals:

1. $$\int \frac{x-9}{(x+5)(x-2)}\ dx$$
2. $$\int \frac{x^2+1}{x^2-x}\ dx$$
3. $$\int \frac{5x^2+3x-2}{x^3+2x^2}\ dx$$
4. $$\int x\ \sin 2x\ \ dx$$
5. $$\int x^2\ \sin x\ \ dx$$
6. $$\int \frac{\ln(2x-1)}{(2x-1)^2}\ dx$$
7. $$\int \cos^3 x\ \sin x\ \ dx$$
8. $$\int \sin^3 x\ \cos^2 x\ \ dx$$
9. $$\int \cos^2 x dx$$
10. $$\int 3\sin^{-1} 2x\ \ dx$$
11. $$\int e^x \cos x\ dx$$
12. $$\int \frac{2x^2+x+1}{x^3+x}\ dx$$
13. $$\int \frac{x+1}{x^3-2x^2+x}\ dx$$

14. $$\int \frac{3x^3 + 2x^2 - 1}{(x-1)^2(x^2+1)}\ dx$$

15. $$\int 5xe^{2x}\ dx$$

b) **Improper integrals.** Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

1. $$\int_1^\infty \frac{1}{x^3}\ dx$$

2. $$\int_{1000}^\infty \frac{1}{x}\ dx$$

3. $$\int_0^\infty xe^{-2x}\ dx$$

4. $$\int_0^\infty \frac{1}{(x+2)(x+3)}\ dx$$

5. $$\int_1^\infty \frac{\ln x}{x^2}\ dx$$

6. Sketch the region $x \geq 3$, $0 \leq y \leq \frac{1}{(x-2)^2}$ and find its area (if the area is finite).

7. Sketch the region $x \geq 0$, $0 \leq y \leq \frac{1}{(x-2)^2}$ and find its area (if the area is finite)

c) **The arc length**.

1. Find the length of the curve $y = x^{3/2}, \quad 1 \leq x \leq 4$
2. Find the length of the curve $y = 4x^{3/2} + 1, \quad 0 \leq x \leq 3$
3. Find the length of the curve $y = \sqrt{1 - x^2}, \quad -1 \leq x \leq 1$
4. Use the Left-Right sum calculator program with $n = 100$ subintervals to approximate the length of the curve $y = e^x$, for $0 \leq x \leq 1$.
5. Use the Left-Right sum calculator program to approximate the length of the curve $y = \sin x$, for $0 \leq x \leq \pi$ to five digits.

d) **The surface area.** Find the area of the surface obtained by rotating the given curve about the specified line.

1. $y = x^3, \quad 0 \leq x \leq 2$ about the x-axis.
2. $y = \sqrt{x}, \quad 4 \leq x \leq 9$ about the x-axis.
3. $y = x^2, \quad 1 \leq x \leq 2$ about the y-axis.
4. $y = 2 + x, \quad 0 \leq x \leq 3$ about the y-axis.

5. Use the Left-Right sum calculator program to approximate the surface area obtained by rotating the curve $y = \sin x$, for $0 \leq x \leq \pi$ about x-axis to four digits.
6. Use the Left-Right sum calculator program with 100 subintervals to find the Right sum which approximates the surface area of the surface obtained by rotating $y = \ln(x^3 + 1)$ $0 \leq x \leq 1$, about y-axis.

e) **Approximate integration**.

1. A CAT scan produces equally spaced cross-sectional views of a human organ that provide information about the organ otherwise obtained only by surgery. Suppose that a CAT scan of a human liver shows cross-sections spaced 2 cm apart. The liver is 12cm long and the cross-sectional areas, in square centimeters are 0, 58, 94, 106, 117, 63, 0. Use the Simpson's Sum to approximate the volume of the liver.
2. A chemical reaction produces a compound X with a rate of 23, 19, 12, 11, 9, 5, 2 liters per second at time intervals spaced by 1 second. Approximate the total volume of the compound X produced in the 6 seconds for which the rate is given using the Simpson's Sum.
3. A 5 mg bolus of dye is injected into the right atrium. The concentration of dye (mg/l) is measured in the aorta at one-second intervals as shown.

t	0	1	2	3	4	5	6	7	8	9	10
$c(t)$	0	.4	2.8	6.5	9.8	8.9	6.1	4	2.3	1.1	0

   Use Simpson's Rule to estimate the cardiac output.
4. A 10 mg bolus of dye is injected into the right atrium. The concentration of dye (mg/l) is measured in the aorta at one-second intervals as shown.

t	0	1	2	3	4	5	6	7	8
$c(t)$	0	0.5	2.4	6.1	8.3	6.3	4.1	1.6	0.4

   Use Trapezoidal Rule to estimate the cardiac output.

f) **Center of Mass.**

1. Find the center of mass of the region bounded by $y = \sin 2x$, $y = 0$, $x = 0$, $x = \pi/2$.
2. Find the center of mass of the region bounded by $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi/4$.

Review 2 – Solutions

More detailed solutions of most of the problems can be found on class handouts.

a) Integration

1. $2\ln|x+5| - \ln|x-2| + c$
2. $x - \ln|x| + 2\ln|x-1| + c$
3. $\frac{1}{x} + 2\ln|x| + 3\ln|2+x| + c$
4. $-\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x + c$
5. $-x^2\cos x + 2x\sin x + 2\cos x + c$
6. $\frac{-\ln(2x-1)}{2(2x-1)} - \frac{1}{2(2x-1)} + c$
7. $\frac{-1}{4}\cos^4 x + c$
8. $\frac{-1}{3}\cos^3 x + \frac{1}{5}\ \cos^5 x + c$
9. $\frac{1}{2}x + \frac{1}{4}\sin 2x + c$
10. $3x\sin^{-1} 2x + \frac{3}{2}\sqrt{1-4x^2} + c$
11. $\frac{1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + c$
12. $\ln x + \frac{1}{2}\ln(1+x^2) + \tan^{-1} x + c$
13. $\ln x - \frac{2}{x-1} - \ln|x-1| + c$
14. $\frac{-2}{x-1} + \frac{9}{2}\ln|x-1| - \frac{3}{4}\ln(1+x^2) + \frac{3}{2}\tan^{-1} x + c$
15. $\frac{5x}{2}e^{2x} - \frac{5}{4}e^{2x} + c$

b) Improper integrals. 1. Convergent, $\frac{1}{2}$ 2. Divergent 3. Convergent, $\frac{1}{4}$ 4. Convergent, $\ln\frac{3}{2} = \ln 3 - \ln 2 = 0.405$ 5. Convergent, 1. 6. Area = 1 7. Divergent, so the area is not finite.

c) The arc length. 1. 7.6337 2. 21.0554 3. π 4. 2.00 5. 3.8202

d) The surface area. 1. 203.0436 2. 81.1418 3. 30.8465 4. 39.986 5. 14.42 6. 4.54

e) Approximate integration. 1. 886.67 cm^3 2. 69 liters 3. .12 liters per second = 7.2 liter per minute 4. .34 liters per second = 20.4 liters per minute

f) Center of Mass. 1. $(\pi/4, \pi/8)$ 2. (.267, .6035)