Calculus 2
Lia Vas

Review for Exam 2

a) **Integration.** Evaluate the integrals:

1. $$\int \frac{x-9}{x^2+3x-10}\ dx$$
2. $$\int 2x\ \sin 3x\ \ dx$$
3. $$\int \frac{\ln(2x-1)}{(2x-1)^2}\ \ dx$$
4. $$\int_1^\infty \frac{1}{x^2}\, dx$$
5. $$\int \frac{5x^2+3x-2}{x^3+2x^2}\ \ dx$$
6. $$\int \frac{2x^2+x+1}{x^3+x}\ \ dx$$
7. $$\int_0^\infty xe^{-2x}\ dx$$
8. $$\int \sin^{10} x\ \cos x\ \ dx$$
9. $$\int \sin^3 x\ \cos^2 x\ \ dx$$
10. $$\int 3\sin^{-1} 2x\ \ dx$$
11. $$\int e^x \sin x\ dx$$
12. $$\int \frac{x^2+1}{x^2-x}\ \ dx$$
13. $$\int x^2\ e^x\ \ dx$$

14. $$\int \sin^2 x dx$$

15. $$\int \sin^5 x dx$$

16. $$\int \frac{x+1}{x^3-2x^2+x}\ dx$$

17. $$\int \frac{2x^2-4x}{(x-1)^2(x^2+1)}\ dx$$

b) **Unbounded area. Improper integrals.** Sketch the following region and find its area if the area is finite.

1. $x \geq 3, \qquad 0 \leq y \leq \frac{1}{(x-2)^2}$
2. $x \geq 0, \qquad 0 \leq y \leq \frac{1}{(x+2)(x+3)}$
3. $x \geq 1, \qquad 0 \leq y \leq \frac{\ln x}{x^2}$

c) **The arc length**.

1. Find the length of the curve $y = x^{3/2}, \quad 1 \leq x \leq 4$
2. Find the length of the curve $y = \sqrt{1-x^2}, \quad -1 \leq x \leq 1$
3. Use the Left-Right sum calculator program with $n = 100$ subintervals to approximate the length of the curve $y = e^x$, for $0 \leq x \leq 1$.
4. Use the Left-Right sum calculator program to approximate the length of the curve $y = \sin x$, for $0 \leq x \leq \pi$ to five digits.

d) **The surface area.** Find the area of the surface obtained by rotating the given curve about the specified line.

1. $y = x^3, \quad 0 \leq x \leq 2$ about the x-axis.
2. $y = \sqrt{x}, \quad 4 \leq x \leq 9$ about the x-axis.
3. $y = x^2, \quad 1 \leq x \leq 2$ about the y-axis.
4. Use the Left-Right sum calculator program to approximate the surface area obtained by rotating the curve $y = \sin x, \quad$ for $0 \leq x \leq \pi$ about x-axis to four digits.
5. Use the Left-Right sum calculator program with 100 subintervals to find the Right sum which approximates the surface area of the surface obtained by rotating $y = \ln(x^3+1)$ $0 \leq x \leq 1$, about y-axis.

e) **Approximate integration**.

1. A CAT scan produces equally spaced cross-sectional views of a human organ that provide information about the organ otherwise obtained only by surgery. Suppose that a CAT scan of a human liver shows cross-sections spaced 2 cm apart. The liver is 12cm long and the cross-sectional areas, in square centimeters are 0, 58, 94, 106, 117, 63, 0. Use the Trapezoidal Sum to approximate the volume of the liver.
2. A chemical reaction produces a compound X with a rate of 23, 19, 12, 11, 9, 5, 2 liters per second at time intervals spaced by 1 second. Approximate the total volume of the compound X produced in the 6 seconds for which the rate is given using the Simpson's Sum.
3. A 5 mg bolus of dye is injected into the right atrium. The concentration of dye (mg/l) is measured in the aorta at one-second intervals as shown.

t	0	1	2	3	4	5	6	7	8	9	10
$c(t)$	0	.4	2.8	6.5	9.8	8.9	6.1	4	2.3	1.1	0

   Use Simpson's Rule to estimate the cardiac output.
4. A 10 mg bolus of dye is injected into the right atrium. The concentration of dye (mg/l) is measured in the aorta at one-second intervals as shown.

t	0	1	2	3	4	5	6	7	8
$c(t)$	0	0.5	2.4	6.1	8.3	6.3	4.1	1.6	0.4

   Use Trapezoidal Rule to estimate the cardiac output.

f) **Center of Mass.**

1. Find the center of mass of the region bounded by $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi/4$.
2. Find the center of mass of the region bounded by $y = \sin 2x$, $y = 0$, $x = 0$, $x = \pi/2$.

Review 2 – Solutions

The class handouts contain more detailed solutions of these problems.

a) Integration

1. $2\ln|x+5| - \ln|x-2| + c$
2. $\frac{-2}{3}x\cos 3x + \frac{2}{9}\sin 3x + c$
3. $\frac{-\ln(2x-1)}{2(2x-1)} - \frac{1}{2(2x-1)} + c$
4. Convergent, 1
5. $\frac{1}{x} + 2\ln|x| + 3\ln|2+x| + c$
6. $\ln x + \frac{1}{2}\ln(1+x^2) + \tan^{-1} x + c$
7. Convergent, $\frac{1}{4}$
8. $\frac{1}{11}\sin^{11} x + c$
9. $\frac{-1}{3}\cos^3 x + \frac{1}{5}\ \cos^5 x + c$
10. $3x\sin^{-1} 2x + \frac{3}{2}\sqrt{1-4x^2} + c$
11. $\frac{-1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + c$
12. $x - \ln|x| + 2\ln|x-1| + c$
13. $x^2e^x - 2xe^x + 2e^x + c$
14. $\frac{1}{2}x - \frac{1}{4}\sin 2x + c$
15. $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c.$
16. $\ln x - \frac{2}{x-1} - \ln|x-1| + c$
17. $\ln|x-1| + \frac{1}{x-1} - \frac{1}{2}\ln(1+x^2) + 2\tan^{-1} x + c$

b) **Unbounded area. Improper integrals.**

1. $A = \int_3^\infty \frac{1}{(x-2)^2}dx$. This integral is convergent and equal to 1 so $A = 1$.
2. $A = \int_0^\infty \frac{1}{(x+2)(x+3)}dx$ This integral is convergent and equal to $A = \ln\frac{3}{2} \approx 0.405$.
3. $A = \int_1^\infty \frac{\ln x}{x^2}dx$. This integral is convergent and equal to 1 so $A = 1$.

c) The arc length. 1. 7.6337 2. π 3. 2.00 4. 3.8202

d) The surface area. 1. 203.0436 2. 81.1418 3. 30.8465 4. 14.42 5. 4.54

e) Approximate integration. 1. 867 cm^3. 2. 69 liters 3. .12 liters per second 4. .34 liters per second

f) Center of Mass. 1. $A = .414, (\bar{x}, \bar{y}) = (.267, .6035)$ 2. $A = 1, (\bar{x}, \bar{y}) = (\frac{\pi}{4}, \frac{\pi}{8})$