Calculus 3
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Formulas for Exam 2

You can bring the formula sheets for Exam 1 to the second exam as well.

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \lvert x \rvert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Rules of Differentiation.**

   a) Product rule: If $y = f \cdot g$, then $y' = f' \cdot g + g' \cdot f$

   b) Quotient rule: If $y = \frac{f}{g}$, then $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$

   c) Chain rule: If $y = f(g(x))$, then $y' = f'(g(x)) \cdot g'(x)$

4. **Integration by parts.** $\int u \; dv = u\, v - \int v\, du$

5. **Maximum and minimum values** of $z = f(x, y)$.

   (a) Find the first partial derivatives f_x and f_y. Set them to 0 and solve for x and y. That gives you **critical points** (a, b).

   (b) Find the second partial derivatives f_{xx}, f_{xy}, f_{yx} and f_{yy}, and the **determinant** D

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

   (c) Then,

      a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local **minimum**.

      b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local **maximum**.

      c) If $D < 0$, then $f(a, b)$ is **not** a local minimum or maximum. It is a **saddle point**.

6. To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = c$, introduce a new variable λ and a new function

$$F = f(x, y, z) - \lambda \cdot (g(x, y, z) - c)$$

   (a) Find all critical values of F (values x, y, z and λ such that $F_x = 0$, $F_y = 0$, $F_z = 0$ and $F_\lambda = 0$).

   (b) Evaluate f at all points from previous step. The largest of these values is the maximum value of f and the smallest is the minimum value of f.

7. **Double Integrals.**

   - $\int\int_D f(x,y)dxdy = \int_a^b \left(\int_{c(x)}^{d(x)} f(x,y)dy\right) dx$ if $D = \{\ (x,y) \mid a \le x \le b,\ c(x) \le y \le d(x)\ \}$
   - $\int\int_R f(x,y)dxdy = \int_c^d \left(\int_{a(y)}^{b(y)} f(x,y)dx\right) dy$ if $D = \{\ (x,y) \mid c \le y \le d,\ a(y) \le x \le b(y)\ \}$
   - The **area of a region** D in xy-plane is $A(D) = \int\int_D dxdy$
   - **Polar Coordinates.**

$$x = r\cos\theta, \quad y = r\sin\theta \qquad \Rightarrow \qquad x^2 + y^2 = r^2, \quad dx\,dy = r\,dr\,d\theta$$

$$\int\int_D f(x,y)\,dx\,dy = \int\int_D f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

8. **Volume.** The volume of the solid region E is

$$V(E) = \int\int\int_E dx\ dy\ dz = \int\int_D (h(x,y) - g(x,y))\ dx\ dy$$

   where D is the projection of E onto the xy-plane.

9. **Surface Area.** The surface area of $z = z(x,y)$ over a region D is

$$S = \int\int_D \sqrt{z_x^2 + z_y^2 + 1}\ \ dx\ dy$$

10. **Applications.**

   - The **average value** of function $f(x,y)$ over the plane region D is

$$f\text{ave} = \frac{1}{A(D)} \int\int_D f(x,y)\ dx\ dy$$

     where $A(D) = \int\int_D dxdy$ is the area of the region D.
   - If a lamina occupies region D in the xy-plane and has density $\rho(x,y)$, then the mass m and the **center of mass** $(\overline{x}, \overline{y})$ are given by

$$m = \int\int_D \rho(x,y)\ dx\ dy \qquad \overline{x} = \frac{1}{m}\int\int_D x\ \rho(x,y)\ dx\ dy \qquad \overline{y} = \frac{1}{m}\int\int_D y\ \rho(x,y)\ dx\ dy$$