Calculus 3
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Formulas for Exam 2

You can bring the formula sheets for Exam 1 to the second exam as well.

1. **Maximum and minimum values** of $z = f(x, y)$.
   (a) Find the first partial derivatives f_x and f_y. Set them to 0 and solve for x and y. That gives you **critical points** (a, b).
   (b) Find the second partial derivatives f_{xx}, f_{xy}, f_{yx} and f_{yy}, and the **determinant** D
   $$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$
   (c) Then,
   a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local **minimum**.
   b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local **maximum**.
   c) If $D < 0$, then $f(a, b)$ is **not** a local minimum or maximum. It is a **saddle point**.

2. To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = c$, introduce a new variable λ and a new function
   $$F = f(x, y, z) - \lambda \cdot (g(x, y, z) - c)$$
   (a) Find all critical values of F (values x, y, z and λ such that $F_x = 0$, $F_y = 0$, $F_z = 0$ and $F_\lambda = 0$).
   (b) Evaluate f at all points from previous step. The largest of these values is the maximum value of f and the smallest is the minimum value of f.

3. **Double Integrals**
   - $\int\int_D f(x, y)dxdy = \int_a^b \left(\int_{c(x)}^{d(x)} f(x, y)dy\right) dx$ if $D = \{\ (x, y) \mid a \le x \le b,\ c(x) \le y \le d(x)\ \}$
   - $\int\int_R f(x, y)dxdy = \int_c^d \left(\int_{a(y)}^{b(y)} f(x, y)dx\right) dy$ if $D = \{\ (x, y) \mid c \le y \le d,\ a(y) \le x \le b(y)\ \}$
   - The **area of a region** D in xy-plane is $A(D) = \int\int_D dxdy$
   - **Polar Coordinates:**
   $$\int\int_D f(x, y)dxdy = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos\theta, r\sin\theta) \cdot r dr\, d\theta$$
   for $D = \{\ (r, \theta) \mid \alpha \le \theta \le \beta,\ \ r_1(\theta) \le r \le r_2(\theta)\ \}$

4. **The volume.** The volume of the solid region E is
   $$V(E) = \int\int\int_E dx\ dy\ dz = \int\int_D (h(x, y) - g(x, y))\ dx\ dy$$
   where D is the projection of E onto the xy-plane.

5. **Surface Area** The surface area of $z = z(x, y)$ over a region D is

$$S = \int\int_D \sqrt{z_x^2 + z_y^2 + 1}\ \ dx\ dy$$

6. **Applications.**

   - The **average value** of function $f(x, y)$ over the plane region D is

     $$f\text{ave} = \frac{1}{A(D)} \int\int_D f(x, y)\ dx\ dy$$

     where $A(D)$ is the area of the region D.

   - If a lamina occupies region D in the xy-plane and has density $\rho(x, y)$, then the mass m and the **center of mass** $(\overline{x}, \overline{y})$ are given by

     $$m = \int\int_D \rho(x, y)\ dx\ dy \qquad \overline{x} = \frac{1}{m} \int\int_D x\ \rho(x, y)\ dx\ dy \qquad \overline{y} = \frac{1}{m} \int\int_D y\ \rho(x, y)\ dx\ dy$$