Calculus 3
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Review for Exam 2

1. **Maximum and Minimum Values.** Find the maximum and minimum values of f.

   (a) $z = 9 - 2x + 4y - x^2 - 4y^2$ (b) $z = x^2 + y^2 + x^2y + 4$
   (c) $z = \frac{x^2y^2 - 8x + y}{xy}$ (d) $z = xy - 2x - y$ (e) $z = 1 + xy - x - y$

2. **Lagrange Multipliers.** Find the maximum and minimum values of f subject to the given constraint(s).

   (a) $f(x, y) = x^2 - y^2;\quad x^2 + y^2 = 1$ (b) $f(x, y) = x^2y;\quad x^2 + 2y^2 = 6$
   (c) $f(x, y, z) = 2x + 6y + 10z;\quad x^2 + y^2 + z^2 = 35$
   (d) $f(x, y, z) = 3x - y - 3z;\quad x + y - z = 0;\quad x^2 + 2z^2 = 1$

3. **Applications.**

   (a) Find the points on the surface $z^2 = xy + 1$ that are closest to the origin.
   (b) Set up the equations for finding the dimensions of the rectangular box with the largest volume if the total surface area is 64 cm^2.
   (c) A cardboard box without a lid is to have volume of 32,000 cm^3. Set up the equations for finding the dimensions that minimize the amount of cardboard used.
   (d) Three alleles for the blood type A, B and O determine four different blood types A (AA or AO), B (BB or BO), AB and O (OO). The Hardy Weinberg Theorem states that the proportion of individuals in a population who carry two different alleles is $P = 2pq + 2pr + 2qr$, where p, q and r are proportions of A, B and O in the population. Use the fact that $p + q + r = 1$, to find the maximum proportion of individuals with two different alleles in the population.

4. **Double Integrals.**

   (a) $\int\int_D (x + 2y)dxdy$ where $D = \{\ (x, y) \mid 0 \le x \le 1,\ \ 0 \le y \le x^2\ \}$
   (b) $\int\int_D 2xdxdy$ where $D = \{\ (x, y) \mid 0 \le y \le 1,\ \ y \le x \le e^y\ \}$
   (c) $\int\int_D y^3dxdy$ where D is the triangular region with vertices (0, 2), (1, 1) and (3, 2)
   (d) $\int\int_D xdxdy$ where D is the disk with center the origin and radius 5 in the first quadrant.
   (e) $\int\int_D xydxdy$ where D is the region in the first quadrant between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 25$.
   (f) $\int\int_D \frac{1}{\sqrt{x^2+y^2}}dxdy$ where D is the region inside $r = 4\cos\theta$ and outside $r = 2$.
   (g) Find the average value of the function $f(x, y) = 4x$ on the region D between the parabolas $y = x^2 - 2$ and $y = 3x - x^2$.
   (h) Find the mass and the center of mass of the lamina that occupies the triangular region with vertices (0,0), (2,1) and (0,3) and has the density function $\rho(x, y) = x + y$.

5. **The Volume.**

   (a) Find the volume of the solid in the first octant bounded by the surface $z = 9 - y^2$ and the plane $x = 2$.

   (b) Find the volume of the solid bounded by the plane $x + y + z = 1$ in the first octant.

   (c) Find the volume of the solid under the surface $z = xy$ and above the triangle with vertices (1, 1), (4, 1) and (1, 2).

   (d) Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \leq 9$.

   (e) Find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the paraboloid $z = 2 - x^2 - y^2$.

   (f) Find the volume of the solid enclosed by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

6. **Surface Area.**

   (a) Find the area of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.

   (b) Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

   (c) Find the area of the part of the surface $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Solutions

More detailed solutions of the problems can be found on the class handouts.

1. Maximum and Minimum Values. (a) Maximum $f(-1, 1/2) = 11$ (b) Minimum $f(0, 0) = 4$, saddle points $(\pm\sqrt{2}, -1)$ (c) Maximum $f(-1/2, 4) = -6$ (d) Saddle point at $(1, 2)$ (e) Saddle point at $(1, 1)$

2. Lagrange Multipliers. (a) Max. $f(\pm 1, 0) = 1$, min. $f(0, \pm 1) = -1$ (b) Max. $f(\pm 2, 1) = 4$, min. $f(\pm 2, -1) = -4$ (c) Max. $f(1, 3, 5) = 70$, min. $f(-1, -3, -5) = -70$ (d) Max. $f(2/\sqrt{6}, -3/\sqrt{6}, -1/\sqrt{6}) = 12/\sqrt{6} = 2\sqrt{6}$, min. $f(-2/\sqrt{6}, 3/\sqrt{6}, 1/\sqrt{6}) = -12/\sqrt{6} = -2\sqrt{6}$.

3. Applications. (a) $(0, 0, \pm 1)$

   (b) Equations: $yz - 2\lambda y - 2\lambda z = 0$, $xz - 2\lambda x - 2\lambda z = 0$, $xy - 2\lambda x - 2\lambda y = 0$, $2xy + 2yz + 2xz = 64$. If solved, the equations would yield: $x = y = z = 4\sqrt{6}/3$ cm.

   (c) Equations: $y + 2z - yz\lambda = 0$, $x + 2z - xz\lambda = 0$, $2x + 2y - xy\lambda = 0$, $xyz = 32,000$. (If solved, the equations would yield: square base of side $x = y =$40 cm, height $z =$20 cm.)

   (d) Max. value 2/3 (or 66.67% of the population) when $p = q = r = 1/3$.

4. Double Integrals. (a) $\frac{9}{20}$ (b) 2.86 (c) $\frac{147}{20}$ (d) $\frac{125}{3}$ (e) $\frac{609}{8}$ (f) $4\sqrt{3} - 4\pi/3$ (g) 3 (h) mass = 6, center of mass = $(\frac{3}{4}, \frac{3}{2})$

5. The Volume. (a) 36 (b) $\frac{1}{6}$ (c) $\frac{31}{8}$ (d) $\frac{81\pi}{2}$ (e) $\frac{5\pi}{6}$ (f) 162π

6. Surface Area. (a) $3\sqrt{14}$ (b) 3.83 (c) 30.85