

Differential Equations  
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Formulas for Exam 1

1. Derivatives.

$y$	$x^n$	$e^x$	$b^x$	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
$y'$	$nx^{n-1}$	$e^x$	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. Integrals.

$y$	$x^n$	$e^x$	$b^x$	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y dx$	$\frac{1}{n+1}x^{n+1}$	$e^x$	$\frac{1}{\ln b} b^x$	$\ln  x $	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. Rules of Differentiation

(a) Product rule: If  $y = f \cdot g$ , then  $y' = f' \cdot g + g' \cdot f$

(b) Quotient rule: If  $y = \frac{f}{g}$ , then  $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$

(c) Chain rule: If  $y = f(g(x))$ , then  $y' = f'(g(x)) \cdot g'(x)$

4. Integration by parts.  $\int u dv = uv - \int v du$

5. Separable Differential Equation:  $P(x)dx = Q(y)dy$ . Integrate both sides.

6. Linear Differential Equation:  $y' + P(x)y = Q(x)$ . Integrating factor:  $I(x) = e^{\int P(x)dx}$ . After multiplying with  $I(x)$ , left side of the equation is equal to derivative of  $I(x) \cdot y$ .

7. Homogeneous Differential Equation:  $y' = f(\frac{y}{x})$ . Use substitution  $u = \frac{y}{x}$  to reduce to a separable equation.

8. Bernoulli's Differential Equation:  $y' + P(x)y = Q(x)y^n$ . Use substitution  $u = y^{1-n}$  to reduce to a linear equation. In this case  $y = u^{1/(1-n)}$ .

9. Exact Differential Equation:  $Mdx + Ndy = 0$  if  $M_y = N_x$ . Find  $F$  as  $\int Mdx$  and equate  $F_y$  with  $N$ . Solution is of the form  $F = 0$ .

10. Autonomous Differential Equation:  $\frac{dy}{dt} = f(y)$ . Can sketch the solutions without solving it. Find equilibrium solutions and check the sign of  $f(y)$ .

11. Basic Differential Equation Models:

(a) Rate proportional to the size:  $y' = ky$ . Here  $k$  is a proportionality constant.

(b) Total rate equal to the difference of rate in and rate out.

(c) Total force equal to the sum of all acting forces.