**Differential Equations**
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# Formulas for Exam 3

1. Definition of the Laplace Transform of $f(t)$.

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)dt$$

2. Table of Laplace transforms.

| **Function** $f(t)$ | **Laplace transform** $F(s)$ |
|---|---|
| $1$ | $\frac{1}{s}$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $t^n e^{at}$ | $\frac{n!}{(s-a)^{n+1}}$ |
| $\sin at$ | $\frac{a}{s^2+a^2}$ |
| $\cos at$ | $\frac{s}{s^2+a^2}$ |
| $e^{at}\sin bt$ | $\frac{b}{(s-a)^2+b^2}$ |
| $e^{at}\cos bt$ | $\frac{s-a}{(s-a)^2+b^2}$ |
| $y'$ | $s\mathcal{L}[y] - y(0)$ |
| $y^{(n)}$ | $s^n\mathcal{L}[y] - s^{n-1}y(0) - \ldots - sy^{(n-2)}(0) - y^{(n-1)}(0)$ |
| $u_c(t)$ | $e^{-cs}\frac{1}{s}$ |
| $u_c(t)f(t-c)$ | $e^{-cs}F(s)$ |
| $e^{ct}f(t)$ | $F(s-c)$ |
| $\delta(t-c)$ | $e^{-cs}$ |
| $\int_0^t f(t-\tau)g(\tau)d\tau$ | $F(s)G(s)$ |

3. Definitions of step and boxcar functions.

   The **unit step function**

$$u_c(t) = \begin{cases} 1, & t \geq c, \\ 0, & t < c. \end{cases}$$

   The **boxcar function** of height $A$ defined by

$$\begin{cases} A, & a \leq t < b, \\ 0, & t < a \text{ and } t \geq b. \end{cases}$$

   can be represented as $Au_a(t) - Au_b(t)$.

4. The **convolution** $f(t) * g(t)$ is defined as

$$f(t) * g(t) = \int_0^t f(t-\tau)g(\tau)d\tau = \int_0^t f(\tau)g(t-\tau)d\tau$$