Differential Geometry
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Project Topics and Report Instructions

Project Overview. Students should be able to research a topic or learn about a method on their own as well, not just via an extensive classroom instruction. The purpose of this project is to practice this skill. You can choose a topic from the **sample topics list below**. You can also pick a topic that is *not* listed below, but should consult with me to double check it is relevant and extensive enough.

If you end up researching a topic especially thoroughly and beyond the scope of the published material you have been using, you will be able to present your findings during **the USciences Annual Research Day** or even to consider preparing your findings for a publication. You can consult with me about this. Keep in mind that the **Research Day is usually in mid April and the abstract is due in mid March**.

Report Instructions. First you need to decide on the topic and find the material that you may need to cover your topic, then write a project report. The purpose of the report is to practice your skills of effectively and convincingly writing about your methods, results and conclusions obtained.

The report should consist of three parts: the introduction, the main part and the conclusion. Throughout the report, make sure that your sentences are clear and your spelling, grammar and punctuation correct. Avoid long sentences and do not use complicated words when you can communicate something using simple and clear phrases. The technical level of your report should be such that your class peers will be able to understand and follow your arguments.

In the **introduction**, you should state the main goal of the project. You can start with the problem statement and the background of the problem. You should clearly state the project objective. Then you can also give a brief summary of the methods used and conclusions obtained.

In the **main part**, you should list your results and conclusions. You should explain the mathematical methods used, prove the main statement using well supported mathematically correct claims, or explain how the mathematical methods apply to physics, chemistry or other sciences. All mathematical data manipulation, calculations, algebraic or numerical work should be in the main part. Make sure that each step is clear and justified.

Finally, you should list your **conclusions** or data testifying the validity and effectiveness of your methods.

You can turn in your project report any time during the semester but not later than **Monday of the 12th week of the semester.** It is very important that you keep this deadline in mind.

List of Sample Project Topics

Curves

1. General helix and Lancret's theorem (Millman-Parker p. 32).

2. Prove the formulas for Frenet-Serret apparatus for a curve with arbitrary parametrization (Millman-Parker p. 47).

3. Global Theory of Curves. Four Vertex Theorem. Knots (Millman-Parker).

Surfaces

4. Show that the surface area does not depend on the parametrization of a surface (Millman-Parker p. 96).

5. Classification of surfaces with constant curvature (Millman-Parker).

6. Show that principal curvatures are the eigenvalues of the second fundamental form. From this fact, it can be deduced that if $\mathbf{v}_1$ is an eigenvector of the second fundamental form and if $\mathbf{v}_2$ is perpendicular to $\mathbf{v}_1$, then $\mathbf{v}_2$ is an eigenvector as well. Thus, the principal directions are perpendicular (Millman-Parker p. 128).

7. Geodesics. Prove that a curve minimizing the distance between the two points is a geodesic (Millman-Parker or Faber).

8. Parallel transport and Levi-Civita connection (Millman-Parker). Gauss curvature via parallel transport (Faber p 52).

9. Global Theory of Surfaces. Euler characteristic. Gauss-Bonnet theorem (relates the Euler characteristic with the Gaussian curvature, Millman-Parker).

Applications and Generalizations.

10. Three Kepler's laws of planetary motion (Stewart, section 14.4) and their generalizations. Volume of hypersphere (Stewart p. 1052) and n-dimensional manifolds.

11. Exterior product: the cross product in n-dimensional space (liavas.net/files/Tim_paper.pdf).

12. Differential Geometry in Theory of Special and General Relativity (Faber).

13. Rigidity of a pizza slice.

14. The metric of the Big Bang: Friedmann-Lemaître-Robertson-Walker metric.

Tensors

15. Einstein notation and tensors. Covariant and contravariant tensors.

16. Covariant and contravariant tensors. Geometric interpretation of Riemann curvature tensor.