

Formulas for Exam 1

1. Derivatives.

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. Integrals.

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b} b^x$	$\ln x $	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. Vectors.

If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$, then

(a) The **length** of \vec{a} is $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

(b) The **dot product** is $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

(c) The **cross product** is $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

4. Line Integrals.

The **work** done by the force $\mathbf{f} = (P, Q, R)$ in moving the particle along the curve C is the line integral

$$W = \int_C \mathbf{f} \cdot d\mathbf{r} = \int_C Pdx + Qdy + Rdz.$$

5. Coordinate Substitutions.

(a) **Cylindrical Coordinates.**

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z.$$

Then $x^2 + y^2 = r^2$ and the Jacobian is $J = r$.

(b) **Spherical Coordinates.**

$$x = r \cos \theta \sin \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \phi.$$

Then $x^2 + y^2 + z^2 = r^2$ and the Jacobian J is $r^2 \sin \phi$.

6. Parametric Surfaces. Surface and Flux Integrals.

Parametric surface is $\mathbf{r}(u, v) = (x(u, v), y(u, v), z(u, v))$.

(a) The tangent plane has the **normal vector** $\mathbf{r}_u \times \mathbf{r}_v = (x_u, y_u, z_u) \times (x_v, y_v, z_v)$.

$$dS = |\mathbf{r}_u \times \mathbf{r}_v| \, dudv$$

$$d\mathbf{S} = \pm(\mathbf{r}_u \times \mathbf{r}_v) \, dudv$$

(b) The **surface integral** of $f(x, y, z)$ is

$$\int \int_S f(x, y, z) \, dS = \int \int_S f(\mathbf{r}(u, v)) |\mathbf{r}_u \times \mathbf{r}_v| \, dudv$$

If $f = 1$, the integral computes the surface area of the surface $\mathbf{r}(u, v)$

$$S = \int \int_S dS.$$

(c) The **flux integral** of $\mathbf{f} = (P, Q, R)$ is

$$\int \int_S \mathbf{f} \cdot d\mathbf{S} = \int \int_S \mathbf{f} \cdot \mathbf{n} \, dS = \pm \int \int_S \mathbf{f} \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, dudv$$

7. Gradient, Curl, and Divergence.

Gradient operator: $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k}$.

Divergence: $\text{div } \mathbf{f} = \nabla \cdot \mathbf{f} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

Curl: $\text{curl } \mathbf{f} = \nabla \times \mathbf{f} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \vec{k}$

8. Stokes' Theorem.

$$\oint_C \mathbf{f} \cdot d\mathbf{r} = \int \int_S \text{curl } \mathbf{f} \cdot d\mathbf{S}$$

9. Divergence Theorem.

$$\int \int_S \mathbf{f} \cdot d\mathbf{S} = \int \int \int_V \text{div } \mathbf{f} \, dx \, dy \, dz$$