Math Methods 1
Lia Vas

Formulas for Exam 2

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\ dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b}\ b^x$	$\ln\|x\|$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Complex numbers.**

$$z = x + iy = r\cos\theta + ir\sin\theta = re^{i\theta}.$$

where $r = |z| = \sqrt{x^2 + y^2}$.

Euler's formula.

$$\cos t + i \sin t = e^{it}.$$

4. **Analytic functions.** Let $f(z) = f(x + iy) = u(x, y) + iv(x, y)$.

Cauchy-Riemann equations.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Laplace Equations.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

5. **Complex integrals and Laurent Series.**

Cauchy's Theorem. If $f(z)$ is analytic and C is a closed curve, then

$$\oint_C f(z)dz = 0.$$

Cauchy's Integral Formula.

$$f^{(n)}(a) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}}\ dz$$

Laurent Series.

If f is analytic at $z = a$ then

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n \quad \text{where } \ a_n = \frac{f^{(n)}(a)}{n!} = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}}\, dz$$

If $z = a$ is an isolated singularity of f then

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^n \quad \text{where} \quad a_n = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}}\, dz$$

The coefficient a_{-1} is the residue of f at a.

Some elementary function expansions.

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} \qquad \frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \qquad \sin z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!} \qquad \cos z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$$

The Residue Theorem. If $z = a$ is an isolated singularity of $f(z)$ and C is a closed, piecewise smooth, positive oriented curve whose interior contains a, then

$$\oint_C f(z)\, dz = 2\pi i a_{-1} = 2\pi i (\text{ coefficient of the term with} \frac{1}{z-a}).$$

If the interior of the curve C contains the isolated singularities $z_1, z_2, \ldots z_n$ of f and $R_1, R_2, \ldots R_n$ are the residues at $z_1, z_2, \ldots z_n$, then

$$\oint_C f(z)\, dz = 2\pi i (R_1 + R_2 + \ldots + R_n).$$

The residue at a pole $z = a$ of order n.

$$a_{-1} = \frac{1}{(n-1)!} \lim_{z \to a} \frac{d^{n-1}}{dz^{n-1}} \left((z-a)^n f(z) \right).$$