Math Methods 1
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Formulas for Exam 3

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1} x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \lvert x \rvert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Integration by parts** $\int u\, dv = u\, v - \int v\, du$

4. **Fourier Series**.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{2\pi n x}{T} + b_n \sin \frac{2\pi n x}{T})$$

$$a_n = \frac{2}{T} \int_{x_0}^{x_0+T} f(x) \cos \frac{2n\pi x}{T}\, dx \qquad b_n = \frac{2}{T} \int_{x_0}^{x_0+T} f(x) \sin \frac{2n\pi x}{T}\, dx$$

If the interval $(x_0, x_0 + T)$ is of the form $(-L, L)$ (thus $T = 2L$), the following holds.

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L}\, dx \qquad b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}\, dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L})$$

5. **Symmetry considerations.** If $f(x)$ is even and defined on $(-L, L)$, then

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L}\, dx, \quad b_n = 0, \text{ and } \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}.$$

If $f(x)$ is odd and defined on $(-L, L)$, then

$$a_n = 0, \quad b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L}\, dx, \quad \text{and} \quad f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}.$$

6. **Complex Fourier Series**.

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{2n\pi i x}{T}} = \sum_{n=-\infty}^{\infty} c_n \left(\cos \frac{2n\pi x}{T} + i \sin \frac{2n\pi x}{T} \right) \text{ where } c_n = \frac{1}{T} \int_{x_0}^{x_0+T} f(x) e^{\frac{-2n\pi i x}{T}} dx.$$

If $f(x)$ is a real function, $c_n = \frac{1}{2}(a_n - ib_n)$ and $c_{-n} = \frac{1}{2}(a_n + ib_n)$ for $n > 0$.

Parseval's Theorem.

$$\frac{1}{T} \int_{x_0}^{x_0+T} |f(x)|^2 dx = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

Parseval's Theorem with Symmetry Considerations. If $f(x)$ is either even or odd function defined on $(-L, L)$,

$$\frac{1}{L} \int_0^L |f(x)|^2 dx = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

7. **Fourier Transform.**

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Inverse Fourier transform.

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

Symmetry considerations. If $f(t)$ is even, $F(\omega) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(t) \cos \omega t \, dt$.

If $F(\omega)$ is even, $f(t) = \sqrt{\frac{2}{\pi}} \int_0^\infty F(\omega) \cos \omega t \, d\omega$.

If $f(t)$ odd, $F(\omega) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(t) \sin \omega t \, dt$.

If $F(\omega)$ is odd, $f(t) = \sqrt{\frac{2}{\pi}} \int_0^\infty F(\omega) \sin \omega t \, d\omega$.

8. **Series Solutions. Regular Point.** If $x = x_0$ is a **regular point** of $y'' + p(x)y' + q(x)y = 0$ a solution can be represented as

$$y = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

When finding the closed form of the solutions, the following may be useful.

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

The series for $e^x, \sin x$ and $\cos x$ converge for every value of x and the series for $\frac{1}{1-x}$ converges on $(-1, 1)$.