Math Methods 1
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Formulas for Exam 4

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \lvert x\rvert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Series Solutions. Regular-Singular Point.** If $x = x_0$ is a **regular-singular point** of $x^2y'' + x\overline{p}y' + \overline{q}y = 0$, a solution can be represented as

$$y = x^r \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n x^{n+r}$$

Form of the solutions in three cases depending on the indical equation.

Case 1. The difference $r_1 - r_2$ is not an integer. In this case, the two linearly independent solutions y_1 and y_2 are given by

$$y_1 = x^{r_1} \sum_{n=0}^{\infty} a_n x^n \quad \text{and} \quad y_2 = x^{r_2} \sum_{n=0}^{\infty} b_n x^n.$$

Case 2. The difference $r_1 - r_2$ is a nonzero integer. If r_1 is the larger root, the two linearly independent solutions y_1 and y_2 are given by

$$y_1 = x^{r_1} \sum_{n=0}^{\infty} a_n x^n \quad \text{and} \quad y_2 = cy_1 \ln x + x^{r_2} \sum_{n=0}^{\infty} b_n x^n.$$

Case 3. The difference $r_1 - r_2$ is zero. In this case, the two linearly independent solutions y_1 and y_2 are given by

$$y_1 = x^{r_1} \sum_{n=0}^{\infty} a_n x^n \quad \text{and} \quad y_2 = y_1 \ln x + x^{r_1+1} \sum_{n=0}^{\infty} b_n x^n.$$

The general solutions is $y = c_1y_1 + c_2y_2$.

When finding the closed form of the solutions, the following may be useful.

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!}\, x^n \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\, x^{2n+1} \qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\, x^{2n}$$

The series for $e^x, \sin x$ and $\cos x$ converge for every value of x and the series for $\frac{1}{1-x}$ converges on $(-1, 1)$.

4. **Groups.** Group axioms. A group is a nonempty set G with operation $\cdot$ such that

   A1. If a, b are elements of G, then $a \cdot b$ is also an element of G, i.e. the operation is **closed**.

   A2. The operation is **associative**: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for any a, b, c in G.

   A3. There is an **identity element** 1 so that $a \cdot 1 = 1 \cdot a = a$ for every element a of G.

   A4. Every element a has the **inverse** a^{-1}, i.e. $a \cdot a^{-1} = a^{-1} \cdot a = 1$.

5. **Classes of groups and their presentations.**

   (a)

Group	notation	no. of el.	presentation
Cyclic of order n	C_n	n	$\langle a \mid a^n = 1 \rangle$
Product of two cyclic	$C_n \times C_m$	mn	$\langle a, b \mid a^n = b^m = 1, ba = ab \rangle$
Dihedral	D_n	$2n$	$\langle a, b \mid a^n = b^2 = 1, ba = a^{n-1}b \rangle$
Product of D_n and C_m	$D_n \times C_m$	$2nm$	$\langle a, b, c \mid a^n = b^2 = c^m = 1, ba = a^{n-1}b, ca = ac, bc = cb \rangle$

   (b) Direct product of finite cyclic groups C_m and C_n:

   $C_m \times C_n$ is isomorphic to C_{mn} if and only if m and n are relatively prime

   Relatively prime = no common factor larger than 1.

   (c) Symmetric group S_n contains all permutations of the set $\{1, 2, \ldots, n\}$. It has $n!$ elements.

6. **Point groups.**

Chem.	Math.	no. of el.	presentation
C_n	C_n	n	$\langle a \mid a^n = 1 \rangle$
C_{nh}	$C_n \times C_2$	$2n$	$\langle a, b \mid a^n = b^2 = 1, ba = ab \rangle$
C_{nv}	D_n	$2n$	$\langle a, b \mid a^n = b^2 = 1, ba = a^{n-1}b \rangle$
C_i, C_s	C_2	2	$\langle b \mid b^2 = 1 \rangle$
D_n	D_n	$2n$	$\langle a, b \mid a^n = b^2 = 1, ba = a^{n-1}b \rangle$
D_{nh}	$C_{nv} \times C_2 = D_n \times C_2$	$4n$	$\langle a, b, c \mid a^n = b^2 = c^2 = 1, ba = a^{n-1}b, ca = ac, bc = cb \rangle$
D_{nd}	D_{2n}	$4n$	$\langle a, b \mid a^{2n} = 1, b^2 = 1, ba = a^{2n-1}b \rangle$
S_{2n}	C_{2n}	$2n$	$\langle a \mid a^{2n} = 1 \rangle$
I	A_5	60	$\langle a, b \mid a^2 = b^3 = (ab)^5 = 1 \rangle$
I_h	$A_5 \times C_2$	120	$\langle a, b, c \mid a^2 = b^3 = (ab)^5 = 1, ac = ca, bc = cb \rangle$
O	S_4	24	$\langle a, b \mid a^2 = b^3 = (ab)^4 = 1 \rangle$
O_h	$S_4 \times C_2$	48	$\langle a, b, c \mid a^2 = b^3 = (ab)^4 = 1, ac = ca, bc = cb \rangle$
T	A_4	12	$\langle a, b \mid a^2 = b^3 = (ab)^3 = 1 \rangle$
T_h	$A_4 \times C_2$	24	$\langle a, b, c \mid a^2 = b^3 = (ab)^3 = 1, ac = ca, bc = cb \rangle$
T_d	S_4	24	$\langle a, b \mid a^2 = b^3 = (ab)^4 = 1 \rangle$
C_∞	$C_\infty = SO(2, R)$	∞	no finite presentation
$C_{\infty v}$	D_∞	∞	no finite presentation
$C_{\infty h}$	$C_\infty \times C_2$	∞	no finite presentation
D_∞	D_∞	∞	no finite presentation
$D_{\infty h}$	$D_\infty \times C_2$	∞	no finite presentation