Math Methods
Lia Vas

Project Topics and Report Instructions

You can turn in your project report any time during the semester but no later than **Tuesday April 11, 2023.**

Project overview. Students should be able to learn about a topic or a method on their own, not just via an extensive classroom instruction. The purpose of this project is to practice this skill. You can choose a topic from the **sample topics list below**. You can also pick a topic that is not listed below, but should consult with me to double check that it is relevant and extensive enough.

The topics below can be divided into two categories:

1. Topics concentrated on applications of mathematics to physics or related sciences. Any of these topics should deepen your understanding of applications of certain mathematical method but should contain some level of mathematical manipulations, mathematical arguments or fragments of mathematical proof.

2. Topics concentrated on purely mathematical arguments demonstrating more details related to a concept covered in class or containing a proof of a statement used, but not proved, in classes. Any of these topics should enhance your ability to follow, understand and adopt arguments involved in mathematical proofs.

If you end up researching a topic especially thoroughly and beyond the scope of the material you used, you could consider presenting your findings during *the SJU Annual Research Day* or even to consider preparing your findings for a publication. You can consult with me about this.

Report instructions. Start with picking a topic and then finding some material that you may need to cover it. You can use only the material referenced or you can use any supplementary material. After that, start writing your project report. The purpose of the report is to practice your skills of effectively and convincingly writing about your methods, results and conclusions obtained.

The report should consist of three parts: the introduction, the main part and the conclusion. Throughout the report, make sure that your sentences are clear and your spelling, grammar and punctuation correct. Avoid long sentences and do not use complicated words when you can communicate something using simple and clear phrases. The technical level of your report should be such that your class peers will be able to understand and follow your arguments.

In the **introduction**, you should state the main goal of the project. You can start with the problem statement and the background of the problem. You should clearly state the project objective. Then you can also give a brief summary of the methods used and conclusions obtained.

In the **main part**, you should expand on the topic, include all the relevant details, mathematical arguments and your conclusions based on them. You should explain the mathematical methods used, prove the main statement using well supported mathematically correct claims, or explain how the mathematical methods apply to physics, chemistry or other sciences. All mathematical data manipulation, calculations, algebraic or numerical work should be in the main part. Make sure that each step is clear and justified.

Finally, you should list your **conclusions** and summarize your project.

The revised version. After you turn the project in, you will receive my feedback on it and you will be able to revise it. The project grade will be based on the **revised version**.

List of some possible project topics

I. Surface Integrals. Stokes' and Divergence Theorems.

1. **Green's Theorem**. Prove the theorem and illustrate its 3-fold use (calculating a line integral via an appropriate double integral, vice versa, and testing the path independence of a vector field) use with examples. Related material can be found in section 11.3 of the Riley-Hobson-Bence textbook and in my Calculus 3 handouts.

2. **General curvilinear coordinates.** Related material is on the last two pages of the "Parametric Surfaces" handout and in section 10.10 of Riley-Hobson-Bence textbook. Present this material and use it to show the formulas for divergence and curl in curvilinear coordinates.

3. **Physics applications of Stokes' and Divergence Theorems**. Formulate the two theorems and summarize their general use, and then illustrate their use in physics with at least one application for each theorem. Related material can be found in sections 11.8.3 and 11.9.2 of the Riley-Hobson-Bence textbook and in our class handouts.

4. **Parametrization of surfaces of revolution. Surface area of a torus.** This topic is for students who will not take Differential Geometry (MA 430). Relevant material can be found in the MA 430 handout "Surfaces part 2" on my website.

5. **Ruled surfaces and parametrization of the Möbius strip.** This topic is for students who will not take Differential Geometry (MA 430). Relevant material can be found in the MA 430 handout "Surfaces part 2" on my website. Explain what a ruled surface is, present a few examples, then introduce the Möbius strip, reflect on its relevance, and explain how to parametrize it as a ruled surface.

II. Complex Analysis

6. **Elementary complex functions.** Euler formula, complex functions $e^z, \sin z$, $\cos z$ and their inverses. The relevant material can be found in sections 3.3, 3.5, and 24.4 of Riley-Hobson-Bence textbook and it is related to the first 2.5 pages of the handout on complex functions.

7. **Power series expansions of elementary complex functions.** Prove the Uniqueness Theorem explaining why x in power series of real functions can be replaced by z to get Laurent series expansions (the first half of section 24.11 of Riley-Hobson-Bence textbook). Use this theorem to explain the arguments made in class notes (bottom of page 6 and the beginning of page 7, Complex Functions handout).

8. **Cauchy's differentiation and integral formulas**. Prove these two formulas. Related material can be found in sections 24.9 and 24.10 of Riley-Hobson-Bence textbook.

9. **Formula that computes the residue at a pole of order k.** Prove the formula for the residue at a pole of order k that involves the limit. Illustrate the use of this formula with examples. The material for this topic can be found in the second half of section 24.11 of Riley-Hobson-Bence textbook.

10. **The Residue Theorem**. Prove the theorem and illustrate its use with examples. The material for this topic can be found in section 24.12 of Riley-Hobson-Bence textbook.

11. **Physics application – complex potentials**. Explain how logarithm of a complex number is defined, what a complex potential is and how it can be used in physics. Illustrate with examples. The material for this topic can be found in section 25.1 of Riley-Hobson-Bence textbook.

III. Fourier Series and Transform

12. **The Fourier series coefficients and the formula for Fourier transform**. Prove the formulas computing the Fourier Series Coefficients (real and complex) and the Fourier Transform. The proofs have been summarized on the class handouts – you can just add full details to that. Related material is also in sections 12.2 and 12.7 of Riley-Hobson-Bence textbook.

13. **Parseval's Theorem**. Prove this theorem and illustrate its use with examples. The material for this topic can be found in section 12.8 of Riley-Hobson-Bence textbook.

14. **Fourier transform in higher dimensions**. The material for this topic can be found in section 13.1.10 of Riley-Hobson-Bence textbook. Include some examples from this section too.

15. **Physics applications – Heisenberg Uncertainty Principle**. The material for this topic can be found in section 13.1.1 of Riley-Hobson-Bence textbook but you may need to supplement it also with material from other sources.

16. **Physics applications – Fraunhofer Diffraction**. Explain how Fourier Transform can be applied in optics, what the Fraunhofer Diffraction is and how it is related to Fourier Transform (see the formula 13.9). The material for this topic can be found in section 13.1.2 of Riley-Hobson-Bence textbook. Include the example from page 438 or another example you may find in other sources.

IV. Series Solutions of Ordinary Differential Equations

17. **Solutions at a regular-singular point.** This topic is covered in the second part of this course PY/MA 371 Mathematical Methods for the Physical Sciences II. There is a detailed handout with many examples on my website (Math Methods page, below "Project Topics"). In your project you can (1) present a general idea and an overview of the method and (2) present one example (any of the three cases) in detail.

18. **Finding the second solution using the derivative method**. The derivative method is used to obtain the form of the second solution in case that the difference of zeros of index equation is an integer. Explain this method and the formula for the second solution. The material for this topic can be found in section 16.4.2 of Riley-Hobson-Bence textbook.

19. **Linear independence of solutions**. This topics is for students who took Linear Algebra, MA 316. Explain what Wronskian is and how it can be used to determine linear independence of solutions. Then show how Wronskian is used to find a linearly independent solution if one solution of the equation is known. Illustrate your claims with examples. The material for this topic can be found in section 16.1 (before 16.1.1) and 16.4.1.

20. **Polynomial solutions.** Sometimes the solutions of second order linear homogeneous differential equation can be found to be polynomials, not infinite series. This topic focuses on such

cases. The material for this topic can be found in section 16.5 of Riley-Hobson-Bence textbook. Illustrate your claims with examples.

21. **Methods of solving second-order linear equations with non-constant coefficients**. This topic is for students who took Differential Equations, MA 320. At the beginning of your class handout on series solutions, the overview of three methods of solving higher-order equations with non-constant coefficients, Legendre's (and Euler's), Exact and Partially Known Complementary Function equations, was presented. Expand this overview into more complete account of the three methods and illustrate each method with examples. The material for this topic can be found in sections 15.2.1, 15.2.2, and 15.2.3 of Riley-Hobson-Bence textbook.

22. **Physics applications – Stokes' equation**. Material for this topics can be found in sections 25.6 (first two parts) of Riley-Hobson-Bence textbook. Optional addition: you can also include discussion on WKB methods and phase memory (section 25.7).

V. Group Theory. Symmetry Groups

23. **Group axioms and cyclic groups**. For this project topic, you can (1) Prove that a set G is a group (by definition given in class) if and only if the rules A1, A2 and D hold. The outline of the proof of this claim can be found on page 4 of the class handout on groups. (2) Expand on cyclic groups. The handout on my website (Math Methods page, below "Project Topics") for more details.

24. **Groups with small number of elements.** More details can be found on my website (Math Methods page, below "Project Topics").

25. **Cycle graphs of groups.** More details can be found on my website (Math Methods, below "Project Topics").

26. **Symmetric and alternating groups**. Expand on symmetric groups S_n, give definition and list properties of alternating groups A_n. You can start by going over the material on S_n and A_n in the class notes and then supplement it with the material from the web and section 28.4 of Riley-Hobson-Bence textbook.

27. **Representations of groups.** More details can be found on my website (Math Methods, below "Project Topics"). You can also include an outline of the way how the point group representations are used to make conclusions about the structural properties and energy levels of the molecule. A web search can produce sufficient material.

28. **Point groups in Chemistry versus groups in Mathematics**. This topics is for students who took Inorganic Chemistry, CH 431. We have seen that some of the different point groups are, in fact, isomorphic. So, for mathematicians, these groups do not have any significant differences. However, from a chemist's point of view, they are significantly different. For example, C_i and C_s are different for a chemist, but for a mathematician, both are just C_2. Then point groups C_{nv} and D_n are both the same (mathematical) dihedral group of $2n$ elements. Taking in account the differences in geometry of corresponding molecules, explain why such groups are considered different by a chemist. Also, expand on the differences in notation in point groups and mathematics and chemistry. A good places to start are mathworld.wolfram.com.

and wikipedia.org. *Optional addition:* There are several online multimedia programs designed to identify symmetry elements and assign point groups to molecules. One of them, the Point Group Tutorial, can be downloaded at http://www.chemistry.emory.edu/pointgrp/ Demonstrate the use of this or similar computer applications.