

Formulas for Exam 3

1. Derivatives.

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$

2. Rules of Differentiation.

(a) Product rule.

$$\text{If } y = f \cdot g, \text{ then } y' = f' \cdot g + g' \cdot f$$

(b) Quotient rule.

$$\text{If } y = \frac{f}{g}, \text{ then } y' = \frac{f' \cdot g - g' \cdot f}{g^2}$$

(c) Chain rule.

$$\text{If } y = f(g(x)), \text{ then } y' = f'(g(x)) \cdot g'(x)$$

3. **Derivative of the inverse function.** If $f(a) = b$ and f is invertible, then

$$(f^{-1})'(b) = \frac{1}{f'(a)}.$$

4. L'Hôpital's Rule

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

5. Linear Approximation.

$$f(a + dx) \approx f(a) + f'(a)dx \quad \text{or} \quad f(x) \approx f(a) + f'(a)(x - a)$$

6. **Tangent Line.** $y_0 = f(x_0)$, $m = f'(x_0)$

$$y - y_0 = m(x - x_0)$$

7. Average and instantaneous rate of change.

(a) The average rate of change of $f(x)$ over $[a, b]$:

$$\frac{f(b) - f(a)}{b - a}$$

(b) The instantaneous rate of change of $f(x)$ at $x = a$: $f'(a)$.

8. **Applications.** The velocity $v(t) = s'(t) = \frac{ds}{dt}$ and the acceleration $a(t) = v'(t) = \frac{dv}{dt} = s''(t) = \frac{d^2s}{dt^2}$.

9. **Analysis using derivatives.**

critical points = x -values at which f' is zero or not defined.

f is increasing/decreasing $\Leftrightarrow f'$ positive/negative.

candidate inflection points = x -values at which f'' is zero or not defined.

f is concave up/down $\Leftrightarrow f''$ positive/negative $\Leftrightarrow f'$ increasing/decreasing.