Calculus 2
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Formulas for Exam 2

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1} x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \lvert x \rvert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Rules of Differentiation**

   a) Product rule: If $y = f \cdot g$, then $y' = f' \cdot g + g' \cdot f$

   b) Quotient rule: If $y = \frac{f}{g}$, then $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$

   c) Chain rule: If $y = f(g(x))$, then $y' = f'(g(x)) \cdot g'(x)$

4. **Integration by parts.**

$$\int u \; dv \; = \; u \, v \; - \; \int v \; du$$

   a) For integrals with polynomial and e^x, $u =$ polynomial.

   b) For integrals with polynomial and $\sin x$ or $\cos x$, $u =$ polynomial.

   c) $\int e^{ax} \sin bx \; dx$ or $\int e^{ax} \cos bx \; dx$. You can start with $u = e^{ax}$. Use the integration by parts twice.

   d) For integrals with log function, try $u =$ log function.

   e) For integrals with inverse trig. functions, try $u =$ inverse trig. function.

5. **L'Hôpital's Rule**

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

6. **Approximate integration.**

$$\text{Left sum} \; = \; \frac{b-a}{n} \; (f(x_0) + f(x_1) + \ldots + f(x_{n-1}))$$

$$\text{Right sum} \; = \; \frac{b-a}{n} \; (f(x_1) + f(x_2) + \ldots + f(x_n \;))$$

$$\begin{aligned}\text{Trapezoidal sum} &= \frac{b-a}{2n}\ \left(f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)\right)\\ &= \frac{1}{2}(\text{Left sum } + \text{ Right sum})\end{aligned}$$

$$\begin{aligned}\text{Simpson's sum} &= \frac{b-a}{3n}\ \left(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\ldots\right.\\ &\left.\ldots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right)\end{aligned}$$

7. **The arc length.**

$$L=\int_a^b \sqrt{1+(y')^2}\ dx.$$

8. **The surface area.**

- Surface area. Revolving $y=y(x)$ on $[a,b]$, axis of revolution x-axis.

$$S=\int_a^b 2\pi y\sqrt{1+(y')^2}\ dx$$

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9. **Applications.**

- If A is the area of the region bounded by $y=f(x)$ from above and $y=g(x)$ from below for $a\le x\le b$, then the coordinates $(\bar{x},\bar{y})$ of the center of mass are

$$\bar{x}=\frac{1}{A}\int_a^b x(f(x)-g(x))\ dx \qquad \bar{y}=\frac{1}{A}\int_a^b \frac{1}{2}((f(x))^2-(g(x))^2)\ dx$$

- If $c(t)$ is the concentration of the dye.

$$\text{Flux, cardiac output } = \frac{\text{amount of the dye } A}{\int_0^T c(t)dt}.$$

10. **Properties of logarithmic function.**

- $\log_a(x\cdot y)=\log_a x+\log_a y$
- $\log_a \frac{x}{y}=\log_a x-\log_a y$
- $\log_a(x^r)=r\log_a x$
- $\log_a x=\frac{\ln x}{\ln a}$

11. **Trigonometric identities.**

(a) $\sin^2 x+\cos^2 x=1$
(b) $\sin^2 x=\frac{1}{2}(1-\cos 2x)$
(c) $\cos^2 x=\frac{1}{2}(1+\cos 2x)$
(d) $\sin x\cos x=\frac{1}{2}\sin 2x$