Calculus 2
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Formulas for Exam 3 and the Final Exam

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1} x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \vert x \vert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Rules of Differentiation**

   a) Product rule: If $y = f \cdot g$, then $y' = f' \cdot g + g' \cdot f$

   b) Quotient rule: If $y = \frac{f}{g}$, then $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$

   c) Chain rule: If $y = f(g(x))$, then $y' = f'(g(x)) \cdot g'(x)$

4. **Integration by parts.** $\int u\ dv = u\, v - \int v\, du$

   a) For integrals with polynomial and e^x, $u =$ polynomial.

   b) For integrals with polynomial and $\sin x$ or $\cos x$, $u =$ polynomial.

   c) $\int e^{ax} \sin bx\, dx$ or $\int e^{ax} \cos bx\, dx$. You can start with $u = e^{ax}$. Use the integration by parts twice.

   d) For integrals with log function, try $u =$ log function.

   e) For integrals with inverse trig. functions, try $u =$ inverse trig. function.

5. **Area between** $f(x) > 0$ **and** x**-axis for** $a < x < b : \int_a^b f(x)\, dx$

   Area between $f(x)$ **and** $g(x)$, $f(x) > g(x)$**, for** $a < x < b : \int_a^b (f(x) - g(x))\, dx$

6. **The volume of the solid of revolution.**

   - axis of revolution x-axis: $V = \int_a^b \pi(f(x))^2\, dx$ or $V = \int_a^b \pi((f(x))^2 - (g(x))^2)\, dx$
   - axis of revolution y-axis: $V = \int_a^b 2\pi x\ f(x)\, dx$ or $V = \int_a^b 2\pi x\ (f(x) - g(x))\, dx$

7. **The arc length.** $L = \int_a^b \sqrt{1 + (y')^2}\ dx$.

8. **The surface area.**

   - axis of revolution x-axis, revolving $y = y(x)$ on $[a, b] \Rightarrow S = \int_a^b 2\pi y \sqrt{1 + (y')^2}\ dx$
   - axis of revolution y-axis, revolving $y = y(x)$ on $[a, b] \Rightarrow S = \int_a^b 2\pi x \sqrt{1 + (y')^2}\ dx$

9. **Approximate integration.**

   - Left sum $= \frac{b-a}{n}\ (f(x_0) + f(x_1) + \ldots + f(x_{n-1}))$
   - Right sum $= \frac{b-a}{n}\ (f(x_1) + f(x_2) + \ldots + f(x_n))$
   - Trapezoidal sum $= \frac{b-a}{2n}\ (f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)) = \frac{1}{2}(\text{Left} + \text{Right})$
   - Simpson's sum $= \frac{b-a}{3n}\ (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots\ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$

10. **Properties of logarithmic function.**

$$\log_a(x \cdot y) = \log_a x + \log_a y \qquad \log_a \tfrac{x}{y} = \log_a x - \log_a y$$
$$\log_a(x^r) = r \log_a x \qquad \log_a x = \tfrac{\ln x}{\ln a}$$

11. **Properties of trigonometric functions.**

$$\sin^2 x + \cos^2 x = 1 \qquad \sin x \cos x = \tfrac{1}{2} \sin 2x$$
$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$
$$\sin x = a \quad \Rightarrow \quad x_1 = \sin^{-1}(a) \quad \text{and} \quad x_2 = \pi - \sin^{-1}(a)$$
$$\cos x = a \quad \Rightarrow \quad x_1 = \cos^{-1}(a) \quad \text{and} \quad x_2 = -\cos^{-1}(a)$$

12. **L'Hôpital's Rule**

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

13. **Applications.**

- Work $= \int_a^b \text{force}\ dx$. For the spring: force $F = kx$, work $= \int_a^b kx\ dx$.
- Average value: $f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x)\ dx$.
  Average rate of change: $f'_{\text{ave}} = \frac{1}{b-a} \int_a^b f'(x) dx = \frac{f(b)-f(a)}{b-a}$.
- Point-slope equation of a line. $y - y_1 = m(x - x_1)$
- Coordinates $(\bar{x}, \bar{y})$ of the center of mass: $\bar{x} = \frac{1}{A} \int_a^b x(f(x) - g(x))\ dx$
  $\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2}((f(x))^2 - (g(x))^2)\ dx$
- If $c(t)$ is the concentration of the dye. Flux, cardiac output $= \dfrac{\text{amount of the dye } A}{\int_0^T c(t) dt}$.

14. **Linear Differential Equation:** $y' + P(x)y = Q(x)$. Integrating factor: $I(x) = e^{\int P(x) dx}$. After multiplying with $I(x)$, left side of the equation is equal to derivative of $I(x) \cdot y$.

15. **Parametric Curve** $x = x(t)$ and $y = y(t)$, $t_1 \le t \le t_2$.

- Derivative: $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$. Second derivative: $\frac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{y'(t)}{x'(t)}\right)}{x'(t)}$.
- Horizontal tangent: $dy = 0$. Vertical tangent: $dx = 0$.
- Area: $A = \int_a^b y\, dx = \pm \int_{t_1}^{t_2} y(t)\, x'(t)\, dt$
- Arc length: $L = \int_{t_1}^{t_2} \sqrt{(x')^2 + (y')^2} dt$
- Surface area around x-axis: $S = \int_{t_1}^{t_2} 2\pi y \sqrt{(x')^2 + (y')^2} dt$
- Surface area around y-axis: $S = \int_{t_1}^{t_2} 2\pi x \sqrt{(x')^2 + (y')^2} dt$

16. **Polar Coordinates.** $x = r \cos\theta$, $y = r \sin\theta$.

- Relation between Cartesian and polar coordinates: $r = \sqrt{x^2 + y^2}$, $\tan\theta = \frac{y}{x}$.
- If $r = r(\theta)$, then the derivative of $x = r(\theta)\cos\theta$ and $y = r(\theta)\sin\theta$ is $\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)}$.
- Area: $A = \int_\alpha^\beta \frac{1}{2}(r(\theta))^2 d\theta$
- Area between $r = f(\theta)$ and $r = g(\theta)$: $A = \int_\alpha^\beta \frac{1}{2}((f(\theta))^2 - (g(\theta))^2) d\theta$
- Arc length: $L = \int_\alpha^\beta \sqrt{(r(\theta))^2 + (r'(\theta))^2} d\theta$.

17. **Taylor Polynomial at $x = a$ of order n.**

$$f(x) \approx \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!} (x-a)^i = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$