

Matlab Notes for Calculus 2

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1. Introduction to Matlab

The main Matlab window is divided into four (or five, depending on the version) parts:

Menu Bar (at the very top)

Current Folder

Command Window

Workspace

Clicking on “New Script”, which is the very first command in the Menu Bar, the Matlab window opens another part, called the

Editor

In some versions of Matlab, the Editor automatically opens on the start.

All Matlab commands are executed in the **Command Window**. If your code is not longer than one line, you can type it in the Command Window and execute it by pressing “enter”.

The **Editor** is used for more complex code. You can have multiple Editor tabs open if you need to work on more than one set of code at a time. The default file is titled “Untitled” so when you type some code in it, you need to save the file and give it a meaningful name. One benefit of typing the code in the Editor is that you can save it while the code typed in the Command Window cannot be saved, just executed. Hence, you can execute the saved code multiple times while executing a command in the Command Window again requires typing the whole command again.

The Directory (**Current Folder** part) shows your current location. If you want to execute some code typed in the Editor, change the location of your current folder to match the folder where you saved the code from the Editor. The current location of the Directory is listed in the long white bar above the Editor.

You can change your current folder by clicking on the folder icon with a green arrow (see the icons on the right side of the white bar listing your current directory). Note that the directory that is opened when Matlab starts is usually not the directory where your files are saved so you will need to change the current folder.

The **Workspace** lists all variables that have been defined and specifies their type and value.

The **Menu Bar** contains some familiar commands, such as New, Open, and Save. When the Editor is open, the Menu Bar contains also Comment and Run. Comment allows us to write text in the code that is not executed which helps your code be more meaningful to you, the programmer, or to a user. Alternatively, you can type “%” before any text and it will be included as a comment.

2. Algebra and Functions

2.1 Basic Arithmetic

You can use +, -, *, \ and ^ to add, subtract, multiply, divide or raise to a power, respectively. For example if you enter in the Command Window:

```
>> 2^3 - 2*2
```

Matlab calculates the answer:

```
ans = 4
```

If you want to perform further calculations with the value of the answer, you can type **ans** rather than retyping the specific answer value. For example,

```
>> sqrt(ans)
```

```
ans = 2
```

To perform symbolic calculations in Matlab, use **syms** to declare the variables you plan to use. For example, suppose that you need factor x^2-3x+2 . First you need `>> syms x` (you are declaring that x is a variable)

Then you can use the command **factor**.

```
>> factor(x^2-3*x+2)
ans = (x-1)*(x-2)
```

Note that we entered **3*x** to represent 3x in the command above. **Entering * for multiplication is always necessary in Matlab.**

Besides **factor** command, you have **simplify** and **expand**.

Practice problems 1

1. Factor $x^3+3x^2y+3xy^2+y^3$.
2. Simplify $\frac{x^3-8}{x-2}$.
3. Evaluate the following expressions a) $\sin(\pi/6)$ b) $\frac{\sqrt{5}+3}{\sqrt{3}-1}$ c) $\log_2(5)$

Solutions

1. **syms x y** followed by **factor(x^3+3*x^2*y+3*x*y^2+y^3)** gives you **ans=(x+y)^3**
2. **syms x** followed by **simplify((x^3-8)/(x-2))** gives you **ans=x^2+2x+4**
3. Enter **sin(pi/6)**. Then **ans=.5** b) Enter **(sqrt(5)+3)/(sqrt(3)-1)**. Then **ans=7.152** c) Enter **log(5)/log(2)**. Then **ans=2.3219**.

2.2 Solving Equations

For solving equations, you can use the command **solve**.

- Represent the variable you are solving using **syms** command.
- Move every term to the left side of the equation so that the equations of the form $g(x)=h(x)$ become $g(x)-h(x)=0$
- If the term on the left side is $f(x)$ and the equation is $f(x)=0$

the command you want to execute is

solve(f(x))

Note that the left side of the equation is in parenthesis. Thus, the command **solve** has the following form

solve(the left side of the equation if the right side is 0)

For example, to solve the equation $x^3-2x-4=0$, we can use

```
>> solve(x^3-2*x-4)
```

and get the following answer:

```
ans = 2 -1+i -1-i
```

Here i stands for the imaginary number $\sqrt{-1}$. This answer tells us that there is just one real

solution, 2.

The command solve often produces a symbolic answer. For example, let us solve the equation $3x^2-8x+2=0$. When executing

```
>> solve(3*x^2-8*x+2)           Matlab produces
ans =      4/3-10^(1/2)/3      10^(1/2)/3+4/3
```

If we want to get, often more meaningful, numerical answer in the decimal form with, say, three significant digits, we can use the command **vpa**, refer to the previous answer as **ans**, and specify how many digits of the answer we need to see. For example.

```
>> vpa(ans, 3)                  produces
ans =      0.279      2.39
```

The command **vpa** has the general form

vpa(*expression you want to approximate, number of significant digits*)

You can solve more than one equation simultaneously. For example suppose that we need to solve the system $x^2+x+y^2=2$ and $2x-y=2$. We can use:

```
>> syms x y
>> [x,y]=solve(x^2+x+y^2-2, 2*x-y-2)
```

to get the answer

```
x = 1      2/5
y = 0      -6/5
```

meaning that there are two solutions $x=1, y=0$ and $x=2/5, y=-6/5$.

Note that the **[x,y]=** part at the beginning of the command was necessary since without it Matlab produces the answer:

```
ans =      x: [2x1 sym]      y: [2x1 sym]
```

This answer tells us just that the solutions are two values of the pair (x,y) but we do not get the solutions themselves. To get the solution vectors displayed, we must use “**[x,y]=**” before the command **solve**.

Practice problems 2

1. Solve the following equations and express the answers as decimal numbers.

a) $x^3-2x=-5$ b) $\log_2(x^2-9)=4$.

2. Solve $5x+2y+4z=8$, $-3x+y+2z=-7$, $2x+y+z=3$ for x , y and z .

Solutions. 1. a) Enter **solve(x^3-2*x+5)**. The only real answer is **-2.09**.

b) Enter **solve(log(x^2-9)/log(2)-4)**. The answers are **ans= 5, -5**.

2. **[x,y,z]=solve(5*x+2*y+4*z-8, -3*x+y+2*z+7, 2*x+y+z-3)** The answer is **x=2 y=-1 z=0**

2.3 Representing a function

To represent a function given by a formula containing a variable x , start by **syms x** if you have not defined x to be your variable already. If we want to call the function f , the following command defines $f(x)$ to be a function defined by the given formula.

f = @(x) formula defining the function

For example, the following command defines the function x^2+3x-2 .

```
>> f = @(x) x^2+3*x-2  
f(x) = x^2+3*x-2
```

After defining a function, we can evaluate it at a point. For example,

```
>> f(2) produces the answer ans = 8.
```

If a function is simple, it might be faster to evaluate a function at a point simply by typing the value of x directly for x. For example, to evaluate $\sin(x)$ at $x=2$, simply type

```
>> sin(2) and obtain the answer ans = .909297.
```

The following table gives an overview of how most commonly used functions or expressions are represented in Matlab.

function or symbol	representation in MATLAB
e^x	exp(x)
$\ln x$	log(x)
$\log x$	log(x)/log(10)
log. base a of x	log(x)/log(a)
$\sin x$	sin(x)
$\cos x$	cos(x)
arctan(x)	atan(x)
π	pi

As when using the calculator, one must be careful when representing a function. For example,

- $\frac{1}{x(x+6)}$ should be represented as **1/(x*(x+6))** not as **1/x*(x+6)** nor as **1/x(x+6)**,
- $\frac{3}{x^2+5x+6}$ should be represented as **3/(x^2+5*x+6)** not as **3/x^2+5*x+6**,
- e^{5x^2} should be represented as **exp(5*x^2)** not as **e^(5*x^2)**, **exp*(5*x^2)**, **exp(5x^2)** nor as **exp^(5*x^2)**.
- $\ln(x)$ should be represented as **log(x)**, not **ln(x)**.
- $\log_3(x^2)$ should be represented as **log(x^2)/log(3)** not as **log(x)/log(3)*x^2**.

3. Graphing

Let us start by declaring that x is a variable:

```
>> syms x
```

The simplest command in Matlab for graphing is **ezplot**. The command has the following form

ezplot(function)

For example, to graph the function x^2+x+1 , you simply type

>> ezplot(x^2+x+1)

A new window will open and the graph will be displayed. To copy the figure to a text file, you can choose the format in which you want to save the figure file (go to the **File** menu of the figure, choose **Save as**, and then select the location, file name and file type, for example png). You can import the figure into the word file using **Insert** and **Image**.

We can specify the different scale on x and y axis. To do this, the command **axis** is used. It has the following form

axis([X_{min} , X_{max} , Y_{min} , Y_{max}])

This command parallels the commands in menu WINDOW on the TI83 calculators.

For example, to see the above graph between x-values -10 and 10 and y-values 0 and 60, you can enter
>> axis([-10 10 0 60])

Note that the domain of function did not change by command axis. To see the graph on the entire domain (in this case [-10, 10]), add that domain after the function in the command ezplot:

ezplot(function, [X_{min} , X_{max}])

In this case,
>> ezplot(x^2+x+1, [-10, 10])
will give you the desired graph.

For the alternative command for graphics, **plot**, you can find more details by typing **help**.

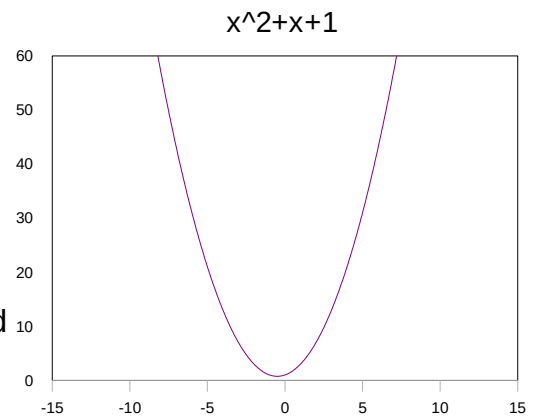
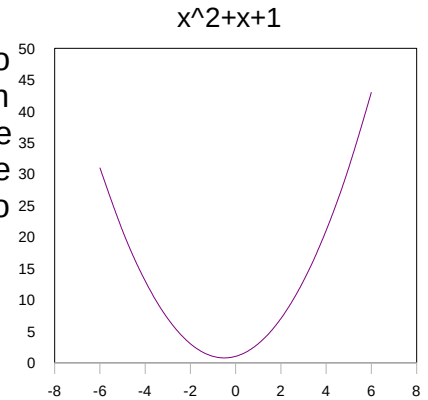
To graph multiple curves on the same plot, you can also use the **ezplot** command. For example to graph the functions $\sin(x)$ and e^{-x^2} , you can use:

>> ezplot(sin(x))
>> hold on
>> ezplot(exp(-x^2))
>> hold off

Practice problems 3

1. Let $f(x) = \frac{x^3 + x + 1}{x}$

a) Represent $f(x)$ as a function in Matlab. Then evaluate it at $x=3$ and $x=-2$.



- b) Find x-value(s) that corresponds to y-value y=2.
 c) Graph f(x) on domain [-4 4].

2. Graph $\ln(x+1)$ and $1-x^2$ on the same plot for x in [-2 6] and y in [-4 4].

Solutions

1. a) `>> f=@(x) (x^3+x+1)/x, >> f(3) ans= 10.333, >>f(-2) ans=4.5.`

b) The problem is asking you to solve equation $\frac{x^3+x+1}{x}=2$. Using solve command, `solve(f(x)-2)`. you get one real answer $x=-1.3247$

c) `ezplot(f(x), [-4,4])`.

2. `>> ezplot(log(x+1)) hold on ezplot(1-x^2) hold off axis([-2, 6, -4, 4])`

4. Solving Equations in Matlab using fzero

In some cases, the command **solve** may fail to produce all the solutions of an equation. In those cases, you can try to find solutions using **fzero** (short for "find zero") command. Just as for **solve**, you need to write equation in the form

$$f(x)=0.$$

So, you need to put all the terms of the equations on one side leaving just zero on the other. To find a solution near the x-value $x=a$, you can use

`fzero('left side of the equation', a)`

The command **fzero**, similarly as **solve** is always followed by expression in parenthesis. Note that the equation should be in single quotes.

If it is not clear what a convenient x-value a should be, you may want to graph the function on the left side of the equation first, check where it intersects the x-axis. Alternatively, you can graph left and right side of the equation that is not in $f(x)=0$ form and see where the two functions intersect. Then decide which x-value you should use.

Example. To solve the equation $e^{x^2}-2=x+4$, we can first graph the functions on the left and right side of the equation using

`syms x ezplot(exp(x^2)-2) hold on ezplot(x+4) hold off`

From the graph, we can see that the two functions intersect at a value near -1 and at a value near 1. To use **fzero**, we need to represent the equation in the form $e^{x^2}-2-(x+4)=0$ (or simplified form $e^{x^2}-x-6=0$). Then, we can find the positive solution by using **fzero** to find a zero near 1 and then to find the negative solution near -1, for example. Thus, both solutions can be obtained by:

```
>> fzero('exp(x^2)-2-(x+4)', 1)      ans = 1.415
>> fzero('exp(x^2)-2-(x+4)', -1)   ans = -1.248
```

Note also that the command `solve(exp(x^2)-2-(x+4))` returns just the positive solution. Thus,

knowing how to use **fzero** command may be really useful in some cases.

5. Calculus

5.1 Differentiation

Start by declaring x for a variable. The command for differentiation is **diff**. It has the following form

diff(function)

For example,

```
>> syms x
```

```
>> diff(x^3-2*x+5)
```

gives us the answer **ans = 3*x^2-2**

To get n -th derivative use

diff(function, n)

For example, to get the second derivative of x^3-2x+5 , use:

```
>> diff(x^3-2*x+5, 2)
```

```
ans = 6*x
```

Similarly, the 23rd derivative of $\sin(x)$ is obtained as follows.

```
>> diff(sin(x), 23)
```

```
ans = -cos(x)
```

To evaluate derivative at a point, we need to represent the derivative as a new function. For example, to find the slope of a tangent line to x^2+3x-2 at point 2, we need to find the derivative and to evaluate it at $x=2$.

```
>> diff(x^2+3*x-2) (first we find the derivative)
```

```
ans = 2*x+3
```

```
>> f = @(x) 2*x+3 (then we representative the derivative as a function)
```

```
>> f(2) (and, finally, we evaluate the derivative at 2)
```

```
ans = 7
```

You can make a derivative (or integral or an output of some other function operator) into a function automatically using **matlabFunction** command using the following format.

f = matlabFunction(*command involving an operation on a given function*)

For example, this last problem can be solved by

```
f = matlabFunction(diff(x^2+3*x-2))
```

followed by **f(2)**.

Practice problems 4

1. Let $f(x) = \frac{x^3+x+1}{x}$ a) Use Matlab to find the first derivative of $f(x)$. b) Evaluate the first

derivative at $x=1$.

2. Let $f(x) = e^{3x^2+1}$. a) Find the first derivative of $f(x)$. b) Find the slope of the tangent line to $f(x)$ at $x=1$. c) Find the critical points of $f(x)$.

3. Find the 12th derivative of the function $(\frac{x}{2}+1)^{65}$.

Solutions.

1. a) `syms x diff((x^3+x+1)/x) ans = 2*x-1/x^2` or `(2*x^3-1)/x^2`.
b) `g=@(x) 2*x-1/x^2`. Then `g(1)` gives you `ans=1`.
2. a) `diff(exp(3*x^2+1)) ans=6*x*exp(3*x^2+1)`
b) `g=@(x) 6*x*exp(3*x^2+1)`. Then evaluate at $x=1$ by `g(1)` and get $6*exp(4)$. To see the answer as a decimal number (say to five nonzero digits) use `vpa(ans, 5)`. Get 327.58.
c) `solve(6*x*exp(3*x^2+1)) ans=0`
3. `diff((x/2+1)^65, 12)`

5.2 Integration

We can use Matlab for computing both definite and indefinite integrals using the command `int`. For the indefinite integrals, start with `syms x` followed by the command

`int(function)`

For example, the command

`>> int(x^2)`

evaluates the integral $\int x^2 dx$ and gives us the answer `ans = 1/3*x^3`

For definite integrals, the command is

`int(function, lower bound, upper bound)`

For example,

`>> int(x^2, 0, 1)`

evaluates the integral $\int_0^1 x^2 dx$ The answer is `ans = 1/3`

In Matlab, `Inf` stands for positive infinity. Matlab can evaluate the improper integrals as well. For example `int(1/x^2, 1, Inf)` produces the answer `ans = 1` telling us that the improper integral considered is convergent. As another example, the command `int(1/x, 0, 1)` produces the answer `ans = inf` telling us that the improper integral considered is divergent.

Matlab can evaluate the definitive integrals of the functions that do not have elementary primitive functions. Recall that the integrals $\int \frac{\sin x}{x} dx$, $\int \frac{e^x}{x} dx$, $\int e^{x^2} dx$

can not be represented via elementary functions. Suppose that we need to find the integral of $\frac{\sin x}{x}$ from 1 to 3. The command `int(sin(x)/x, 1, 3)` does not gives us a numerical value. We

have just `ans = sinint(3)-sinint(1)` Using the command `vpa`, we obtain the answer in numerical form. For example, `vpa(ans, 4)` gives us `ans = 0.9026`.

Practice problems 5

- Evaluate the following integrals. a) $\int x e^{-3x} dx$ b) $\int_0^1 x e^{-3x} dx$.
- Using Matlab, determine if the following integrals converge or diverge. If they converge, evaluate them.

a) $\int_1^{\infty} \frac{3}{x^2+5x+6} dx$ b) $\int_1^2 \frac{1}{(x-1)(x+1)} dx$

Solutions.

- ```

syms x
int(x*exp(-3*x))
ans=-1/3*x*exp(-3*x)-1/9*exp(-3*x)

```
  - ```

int(x*exp(-3*x), 0,1)
ans=-4/9*exp(-3)+1/9
vpa(ans, 4)
ans=.08898

```
- ```

int(3/(x^2+5*x+6), 1, Inf)
vpa(ans, 4)
ans= .8338

```

So, the integral is convergent.
  - ```

int(1/((x-1)*(x+1)), 1, 2)
ans= Inf

```

So, the integral is divergent.

5.3 Limits

You can use **limit** to compute limits, left and right limits as well as infinite limits. For example, to evaluate the limit when $x \rightarrow 2$ of the function $\frac{x^2-4}{x-2}$, we have:

```

>> syms x
>> limit((x^2-4)/(x-2), x, 2)
ans = 4

```

You can also evaluate left and right limits. For example:

```

>> limit(abs(x)/x, x, 0, 'left')
ans = -1
>> limit(abs(x)/x, x, 0, 'right')
ans = 1

```

Limits at infinity:

```

>> limit(exp(-x^2-5)+3, x, Inf)
ans = 3

```

Practice problem 6

Find the limits of the following functions at indicated values.

a) $f(x) = \frac{x^{12}-1}{x^3-1}$, $x \rightarrow 1$ b) $f(x) = 3+e^{-2x}$, $x \rightarrow \infty$ c) $f(x) = \frac{6x^3-4x+5}{2x^3-1}$, $x \rightarrow \infty$

Solutions.

- ```

syms x
limit((x^12-1)/(x^3-1), x, 1)
ans=4

```
- ```

limit(3+exp(-2*x), x, Inf)
ans=3

```
- ```

limit((6*x^3-4*x+5)/(2*x^3-1), x, Inf)
ans=3

```

## 6. Graphic Continued

### 5.1 Parametric Plots

We can use the command **ezplot** to graph a parametric curve as well. For example, to graph a circle  $x = \cos t$ ,  $y = \sin t$  for  $0 \leq t \leq 2\pi$ , we have:

```
>> syms t
>> ezplot(cos(t), sin(t), [0, 2*pi])
```

## 5.2 Special effects

You can change the title above the graph by using the command **title**. For example,

```
>> ezplot(x^2+2*x+1)
>> title 'a parabola'
```

You can add labels to x and y axis by **xlabel** and **ylabel**.

You can produce an animated graph using **comet**. This command produces a parametric plot of a curve just as **ezplot** does, except that you can see the curve being traced out in time. For example, we can trace the motion on the circle  $x = \cos t$ ,  $y = \sin t$  by using

```
>> t = 0:0.1:4*pi; (meaning that t is taking values 0, 0.1, 0.2, 0.3,..., 4π)
>> comet(cos(t), sin(t))
```

If the point is moving too fast, you can reparameterize the same circle as follows, for example.

```
>> t = 0:0.1:200*pi;
>> comet(cos(t/50), sin(t/50))
```

### Practice problems 7

1. Graph the parametric curve  $x = t \cos t$ ,  $y = t \sin t$  for  $0 \leq t \leq 10\pi$ .
2. Trace the curve  $x = t \cos (t/20)$ ,  $y = t \sin (t/20)$  in time for  $0 \leq t \leq 200\pi$ .

### Solutions

```
1. syms t ezplot(t*cos(t), t*sin(t), [0, 10*pi]) 2. t = 0:0.5:200*pi;
comet(t*cos(t/20), t*sin(t/20))
```

## 7. Differential Equations

### 7.1 Basics of Differential Equations

A solution of a differential equation given by a formula is said to be a symbolic solution. Matlab command **dsolve** produces a symbolic solution. In this case, **y(x)** can be declared as a function of x using the command **syms** and the derivative of the function y can be represented by **diff(y, x)**. Note that the order of variables here (dependent first, independent second) matches the order in the expression dy/dx. The equality sign in the equation is represented by **==**. The command **dsolve** has the following form.

**dsolve(equation, independent variable)**

For example, suppose that we want to find the **general solution** of the equation  $x y' - y = 1$ .

You can do that by **syms y(x)** followed by

```
>> dsolve(x*diff(y,x)-y==1, x) producing ans = C1*x-1
```

Thus, the general solution consists of functions of the form  $y = cx - 1$ , where  $c$  is any constant.

If the equation also has an initial condition, one can get the **particular solution** using the following format. The equality sign in the initial condition should also be represented by **==**.

**dsolve(equation, initial condition, independent variable)**

For example, to solve  $x y' - y = 1$  with the initial condition  $y(1)=5$ , the command

```
>> dsolve(x*diff(y,x)-y==1, y(1)==5, x) produces the answer ans = 6*x-1
```

The solution can be represented as a function of  $x$  by

```
>> f(x)=dsolve(x*diff(y,x)-y==1, y(1)==5, x)
```

Such function can be evaluated at an  $x$ -value, 3 for example, simply by **f(3)** producing the  $f$ -value of 17. The particular solution  $f(x)$  can also be graphed by **ezplot(f(x))**.

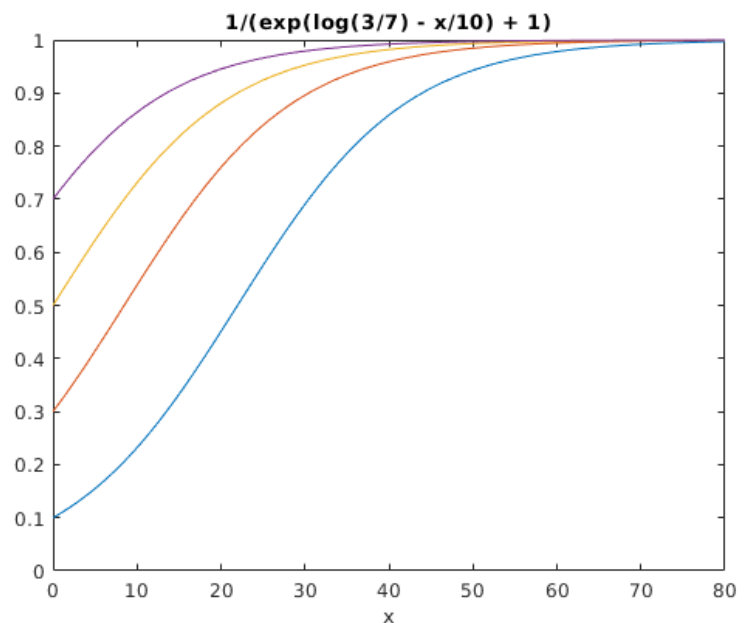
We can graph a couple of different solutions on the same chart by using **hold on** and **hold off** commands. For example, to graph the solutions of differential equation  $y'=0.1y(1-y)$  for several different initial conditions,  $y(0)=0.1$ ,  $y(0)=0.3$ ,  $y(0)=0.5$  and  $y(0)=0.7$ , first find the four particular solutions, then graph them using **hold on** and **hold off** commands. To observe the limiting behavior of the solution, in examples below the interval  $[0, 80]$  is used as the domain.

Find the four solutions as follows.

```
>> syms y(x)
>> eqn = diff(y,x)==0.1*y*(1-y)
>> y1=dsolve(eqn, y(0)==0.1, x)
y1 = 1/(exp(log(9) - x/10) + 1)
>> y2=dsolve(eqn, y(0)==0.3, x)
y2 = 1/(exp(log(7/3) - x/10) + 1)
>> y3=dsolve(eqn, y(0)==0.5, x)
y3 = 1/(exp(-x/10) + 1)
>> y4=dsolve(eqn, y(0)==0.7, x)
y4 = 1/(exp(log(3/7) - x/10) + 1)
```

Graph the four solutions as follows.

```
>> ezplot(y1,[0,80])
>> hold on
>> ezplot(y2,[0,80])
>> ezplot(y3,[0,80])
>> ezplot(y4,[0,80])
>> hold off
>> axis([0,80,0,1])
```



## 7.2 Second Order Equations

Symbolic solutions of the second order equations can be found using **dsolve** similarly as the first order differential equations. The second derivative  $y''$  of  $y(x)$  can be represented by **diff(y, x, 2)**. For example, the general solution of  $y''-3y'+2y = \sin x$  can be found by **syms y(x)** followed by

```
>> dsolve(diff(y, x, 2)-3*diff(y, x)+2*y==sin(x), x)
ans = C1*exp(x) + (10^(1/2)*cos(x - atan(1/3)))/10 + C2*exp(2*x)
```

If the equation has the initial conditions  $y(0) = 1$  and  $y'(0)=-1$ , for example,  $y'$  can be represented as **Dy** and then one can use **Dy(0) == -1** for the second initial condition.

```
>> Dy=diff(y,x)
>> dsolve(diff(y, x, 2)-3*diff(y, x)+2*y==sin(x), y(0)==1, Dy(0)==-1, x)
ans = (5*exp(x))/2 - (9*exp(2*x))/5 + (10^(1/2)*cos(x - atan(1/3)))/10
```

### Practice problems 8

- Find the general solution of the differential equation  $y'-2y=6x$ .
  - Find the particular solution with initial condition  $y(0)=3$ .
  - Plot the particular solution on interval  $[0,2]$  and find the value of this solution at 2.
- Graph the solutions of the differential equation  $y' = x+y$  for the  $y$ -values of the initial condition  $y(0)$  taking integer values between -2 and 4 (thus  $y(0)=-2,-1,0,1,2,3$ , and 4).
- Find the general solution of the equation  $y''-4y'+4y = e^x + x^2$ . Then find the particular solution if the initial values are  $y(0)=8$  and  $y'(0)=3$ .

### Solutions.

1. a) General solution

```
dsolve(diff(y,x)-2*y==6*x, x)
ans=(C1*exp(2*x))/2-3*x- 3/2
```

b) Particular solution

```
f(x)=dsolve(diff(y,x)-2*y==6*x,
y(0)==3, x) f(x)=(9*exp(2*x))/2-3*x- 3/2
```

c) To graph use **ezplot(f(x), [0 2])**. To find the value at 2, use **f(2)** obtain **ans=238.19**

2. Finding seven particular solutions.

```
eqn = diff(y,x)==x+y
```

```
s1 = dsolve(eqn, y(0)==-2, x)
```

```
s2 = dsolve(eqn, y(0)==-1, x) ... etc ..
```

```
s7 = dsolve(eqn, y(0)== 4, x)
```

Then plot them on the same graph.

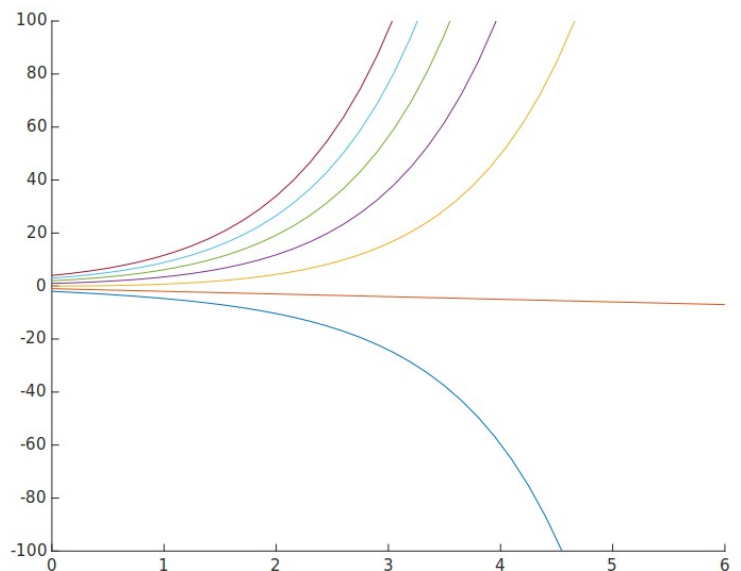
```
hold on
```

```
ezplot(s1)
```

```
ezplot(s2) ...etc...
```

```
ezplot(s7)
```

```
hold off
```



`axis([0 6 -100 100])`

3. `>> dsolve(diff(y,x,2)-4*diff(y,x)+4*y==exp(x)+x^2, x)`

`>> Dy=diff(y,x)`

`>> dsolve(diff(y,x,2)-4*diff(y,x)+4*y==exp(x)^2+x^2, y(0)==8, Dy(0)==3, x)`

`ans = x/2 + (61*exp(2*x))/8 - (51*x*exp(2*x))/4 + (x^2*exp(2*x))/2 + x^2/4 + 3/8`