Calculus 2
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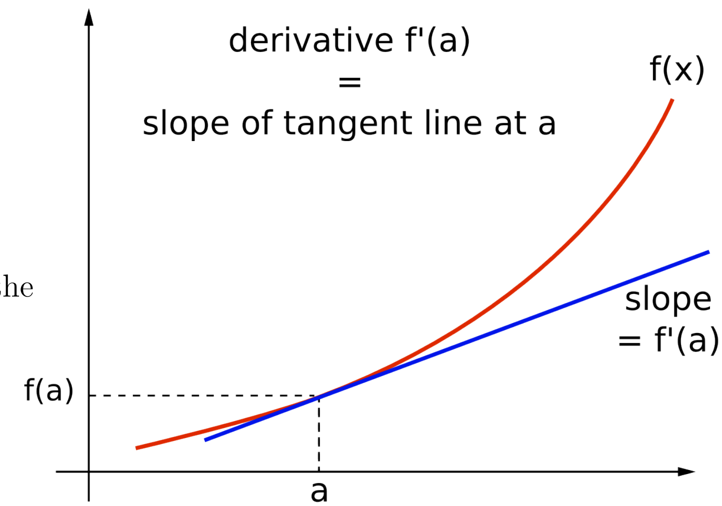
Review of Differentiation

The value of the derivative $f'(x)$
of $f(x)$ at $x = a$ is
the slope of the line **tangent**
to the graph of $f(x)$ at $x = a$.



Thus, an equation of the line **tangent** to the graph of $f(x)$ at $x = a$

1. passes the point $(a, f(a))$ and
2. has the slope $m = f'(a)$.

So, you can find it using the point-slope equation of a line with slope m passing the point (x_0, y_0)

$$y - y_0 = m(x - x_0).$$

Recall also the rules for finding derivative.

The Power Rule	$y = x^n$	$\Rightarrow$	$y' =$	nx^{n-1}	
The Product Rule	$y = fg$	$\Rightarrow$	$y' =$	$f'g + g'f$	
The Quotient Rule	$y = \frac{f}{g}$	$\Rightarrow$	$y' =$	$\frac{f'g - g'f}{g^2}$	
The Chain Rule	$y = f(g(x))$	$\Rightarrow$	$y' =$ derivative of the composite	$f'(g(x))$ $\cdot$ derivative of the outer, keep the inner unchanged	$g'(x)$ derivative of the inner

Practice Problems.

1. **Derivative.** Find the derivative for the given function.

(a) $f(x) = 2x^5 - 3x^3 + 5x - 9$
(b) $f(x) = \frac{x^3}{2} + \sqrt{x^3}$
(c) $f(x) = \frac{4}{x^2} - \frac{1}{3x^6}$
(d) $f(x) = (6x^2 + 5)^{10}$
(e) $f(x) = 2x\sqrt{x^3 + 2}$
(f) $f(x) = \frac{x^2+3x}{5x-2}$
(g) $f(x) = \frac{(x^2+3)^4}{(3x^2+1)^5}$

2. Find an equation of the line tangent to the graph of the given equation at the indicated point.

(a) $f(x) = \frac{x^2+3x-5}{x}$ at $x = 1$.

(b) $f(x) = \frac{2}{x} + \frac{x}{2}$ at $x = 2$.

(c) $f(x) = \sqrt{x^3} + \sqrt[3]{x^2}$ at $x = 1$.

Solutions.

1. (a) $f'(x) = 10x^4 - 9x^2 + 5$ (b) $f'(x) = \frac{3}{2}x^2 + \frac{3}{2}\sqrt{x}$ (c) $f'(x) = \frac{-8}{x^3} + \frac{2}{x^7}$

(d) $f'(x) = 120x(6x^2+5)^9$ (e) $f'(x) = 2\sqrt{x^3+2} + 3x^3(x^3+2)^{-1/2}$

(f) $f'(x) = \frac{(2x+3)(5x-2)-5(x^2+3x)}{(5x-2)^2}$ (g) $f'(x) = \frac{4(x^2+3)^3 2x(3x^2+1)^5 - 5(3x^2+1)^4 6x(x^2+3)^4)}{(3x^2+1)^{10}}$

2. (a) You can find the derivative of $f(x)$ using the quotient rule, or you can simplify $f(x)$ as follows $f(x) = x + 3 - 5x^{-1}$ and differentiate term by term. Obtain that $f'(x) = 1 + \frac{5}{x^2} = \frac{x^2+5}{x^2}$. Plugging $x = 1$ in the derivative, obtain the slope $m = 6$. Since $f(1) = -1$, the line passes the point $(1, -1)$. So, an equation of the tangent line is $y - (-1) = 6(x-1) \Rightarrow y + 1 = 6x - 6 \Rightarrow y = 6x - 7$.

(b) Plugging $x = 2$ in the derivative $f'(x) = -2x^{-2} + \frac{1}{2}$, you obtain the slope $m = \frac{-1}{2} + \frac{1}{2} = 0$. Since $f(2) = 1 + 1 = 2$, the tangent line passes $(2, 2)$. So, an equation of the tangent line is $y - 2 = 0(x-2) \Rightarrow y - 2 = 0 \Rightarrow y = 2$.

(c) Plugging $x = 1$ in the derivative $f'(x) = \frac{3}{2}x^{1/2} + \frac{2}{3}x^{-1/3}$, you obtain the slope $m = \frac{13}{6}$. Since $f(1) = 2$, the tangent line passes $(1, 2)$. So, an equation of the tangent line is $y - 2 = \frac{13}{6}(x-1) \Rightarrow y - 2 = \frac{13}{6}x - \frac{13}{6} \Rightarrow y = \frac{13}{6}x - \frac{13}{6} + \frac{12}{6} \Rightarrow y = \frac{13}{6}x - \frac{1}{6}$.