Calculus 2
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Derivatives and Integrals of Trigonometric and Inverse Trigonometric Functions

Trigonometric Functions.

Recall that

if $y = \sin x$, then $y' = \cos x$ and
if $y = \cos x$, then $y' = -\sin x$.

Thus,

$\int \sin x \, dx = -\cos x + c$ and
$\int \cos x \, dx = \sin x + c$.

The derivatives and integrals of the remaining trigonometric functions can be obtained by expressing these functions in terms of sine or cosine using the following identities: $\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$, $\sec x = \frac{1}{\cos x}$, $\csc x = \frac{1}{\sin x}$.

Example 1. Find derivative of $\tan x$. Simplify your answer.

Solution. Using the formula $\tan x = \frac{\sin x}{\cos x}$ and the quotient rule, obtain $\frac{d \tan x}{dx} = \frac{\cos x \cos x - (-\sin x)\sin x}{\cos^2 x} = \frac{\cos^2 x + \sin^x x}{\cos^2 x} = \frac{1}{\cos^2 x}$ or $\sec^2 x$.

Inverse Trigonometric Functions. The function $\sin x$ passes horizontal line test for $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$ so it has an inverse. The inverse function is denoted by $\sin^{-1} x$ or $\arcsin x$. Since the range of $\sin x$ on $[\frac{-\pi}{2}, \frac{\pi}{2}]$ is [-1,1,], the interval [-1,1] is the domain of $\sin^{-1} x$. We also have the following cancellation rule.

$$\sin(\sin^{-1} x) = x \quad \text{for} \quad -1 \leq x \leq 1 \quad \text{and} \quad \sin^{-1}(\sin x) = x \quad \text{for} \quad \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$$

Similarly, one obtains $\cos^{-1} x$ on $[-1, 1]$ is the inverse of $\cos x$ for $0 \leq x \leq \pi$ and the analogous cancellation rule holds. The function $\tan^{-1} x$ on $(-\infty, \infty)$ has the inverse $\tan x$ for $\frac{-\pi}{2} < x < \frac{\pi}{2}$. The inverses of other trigonometric functions can be obtained similarly.

Careful: when using notation $\sin^{-1} x$ do not mix this function up with $(\sin x)^{-1} = \frac{1}{\sin x}$.

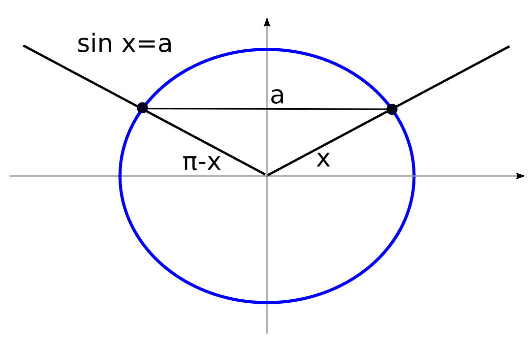
When solving an equation of the form $\sin x = a$ for x where a is a number in $[-1, 1]$, the second cancellation formula implies that $x = \sin^{-1} a$ is one solution of this equation and that this value will be in interval $[\frac{-\pi}{2}, \frac{\pi}{2}]$. In many cases, we need more than this one solution, in particular, the second solution in interval $[0, 2\pi]$.

Using the trigonometric circle (or the graph of sine function) we can see that if

$$x_1 = \sin^{-1} a,$$

the second solution can be obtained as

$$x_2 = \pi - x_1 = \pi - \sin^{-1} a.$$

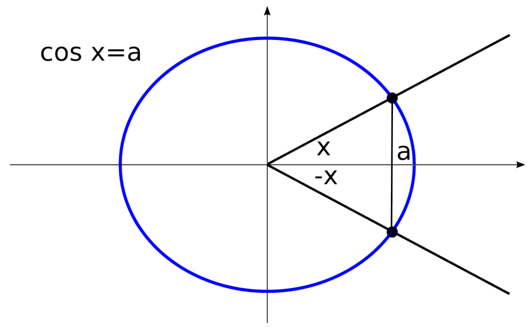


Similarly, when solving $\cos x = a$, in interval $[0, 2\pi]$ the first solution can be found as

$$x_1 = \cos^{-1} a$$

and the second solution can be obtained as

$$x_2 = -x_1 = -\cos^{-1} a.$$

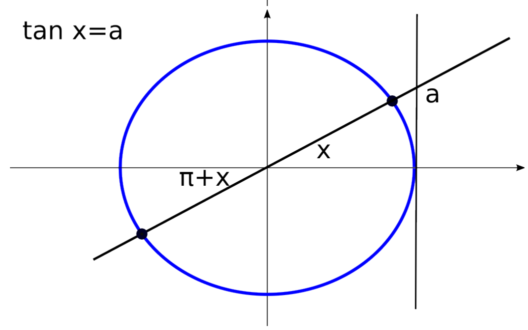


For the equation $\tan x = a$, on $[0, 2\pi]$, the first solution can be found as

$$x_1 = \tan^{-1} a$$

and the second solution can be obtained as

$$x_2 = \pi + x_1 = \pi + \tan^{-1} a.$$



Derivatives of the Inverse Trigonometric Functions.

The formula for the derivative of $y = \sin^{-1} x$ can be obtained using the fact that the derivative of the inverse function $y = f^{-1}(x)$ is the reciprocal of the derivative $x = f(y)$.

$$y = \sin^{-1} x \Rightarrow x = \sin y \Rightarrow x' = \cos y \Rightarrow y' = \frac{1}{x'} = \frac{1}{\cos y} = \frac{1}{\cos(\sin^{-1} x)}.$$

To be able to simplify this last expression, one needs to represent $\cos y$ in terms of $\sin y$. This can be done using the trigonometric identity

$$\sin^2 y + \cos^2 y = 1 \Rightarrow \cos y = \sqrt{1 - \sin^2 y} \Rightarrow \cos(\sin^{-1} x) = \sqrt{1 - (\sin(\sin^{-1} x))^2} = \sqrt{1 - x^2}.$$

Thus, we obtain the formula for the derivative of $y = \sin^{-1} x$ to be

$$y' = \frac{1}{\sqrt{1 - x^2}}$$

Similarly, one obtains the following formulas.

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{x^2 + 1} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2 - 1}}$$

Differentiating the trigonometric identities $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, and $\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}$, we obtain that the derivatives of $\cos^{-1} x$, $\cot^{-1} x$, and $\csc^{-1} x$ are negative of the derivatives of $\sin^{-1} x$, $\tan^{-1} x$, and $\sec^{-1} x$ respectively.

Example 2. Find the derivatives of the following functions.

(a) $y = \sin^{-1}(2x)$ (b) $y = x \tan^{-1} \sqrt{x}$ (c) $y = \sin^{-1} x^2 + \sqrt{1 - x^2}$

Solution. (a) Use the chain rule with $2x$ as the inner function and $\sin^{-1} x$ as the outer. The derivative of the outer with the inner function kept unchanged is $\frac{1}{\sqrt{1-(2x)^2}} = \frac{1}{\sqrt{1-4x^2}}$. The derivative of the inner function is 2 so the derivative of $y = \sin^{-1}(2x)$ is

$$y' = \frac{2}{\sqrt{1-4x^2}}.$$

(b) The function is a product of $f = x$ and $g = \tan^{-1} \sqrt{x}$. Use the chain rule for the derivative of g. Obtain that $f' = 1$ and $g' = \frac{1}{1+(\sqrt{x})^2} \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}(1+x)}$. Hence the derivative of $y = x \tan^{-1} \sqrt{x}$ is

$$y' = f'g + g'f = 1 \cdot \tan^{-1} \sqrt{x} + \frac{1}{2\sqrt{x}(1+x)} \cdot x = \tan^{-1} \sqrt{x} + \frac{\sqrt{x}}{2(1+x)}.$$

(c) The function is a sum of two terms and you can differentiate term by term. Use the chain rule for the first term to get $\frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$. The derivative of the second term is $\frac{1}{2}(1-x^2)^{-1/2}\,(-2x) = -\frac{x}{\sqrt{1-x^2}}$. Hence the derivative of the function $y = \sin^{-1} x^2 + \sqrt{1-x^2}$ is

$$y' = \frac{2x}{\sqrt{1-x^4}} - \frac{x}{\sqrt{1-x^2}}.$$

Integrals producing inverse trigonometric functions. The above formulas for the the derivatives imply the following formulas for the integrals.

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + c$$

$$\int \frac{1}{x^2+1}\, dx = \tan^{-1} x + c \qquad\qquad \int \frac{1}{x\sqrt{x^2-1}}\, dx = \sec^{-1} x + c$$

Example 3. Evaluate the following integrals.

$$\text{(a)} \int \frac{1}{\sqrt{1-9x^2}}\, dx \qquad\qquad \text{(b)} \int \frac{1}{9x^2+1}\, dx$$

Solution. (a) Note that the integrand matches the form $\frac{1}{\sqrt{1-u^2}}$ with $u^2 = 9x^2$. This produces the desired substitution $u^2 = 9x^2 \Rightarrow u = 3x$. Hence $du = 3dx \Rightarrow dx = \frac{du}{3}$. Thus we have that

$$\int \frac{1}{\sqrt{1-9x^2}}\, dx = \int \frac{1}{\sqrt{1-u^2}}\, \frac{du}{3} = \frac{1}{3} \int \frac{1}{\sqrt{1-u^2}}\, du = \frac{1}{3} \sin^{-1}(u) + c = \frac{1}{3} \sin^{-1}(3x) + c.$$

(b) Note that the integrand matches the form $\frac{1}{1+u^2}$ with $u^2 = 9x^2$. This produces the desired substitution $u^2 = 9x^2 \Rightarrow u = 3x$. Hence $du = 3dx \Rightarrow dx = \frac{du}{3}$. Thus we have that

$$\int \frac{1}{9x^2+1}\, dx = \int \frac{1}{u^2+1}\, dx\, \frac{du}{3} = \frac{1}{3} \int \frac{1}{1+u^2}\, du = \frac{1}{3} \tan^{-1}(u) + c = \frac{1}{3} \tan^{-1}(3x) + c.$$

Example 4. Compare the methods for evaluating the following integrals.

$$\text{(a)} \int \frac{x}{\sqrt{1-9x^2}}\, dx \qquad \text{(b)} \int \frac{1}{\sqrt{1-9x^2}}\, dx$$

Use your conclusion to determine the method for evaluating the integral

$$\text{(c)} \int \frac{x+1}{\sqrt{1-9x^2}}\, dx$$

Solution. (a) Although the denominator matches the one from the previous problem, the substitution $u = 3x$ would not work because of the x in the numerator. However, the substitution $u = 1-9x^2$ has $du = -18x dx \Rightarrow dx = \frac{du}{-18x}$ so the x term from du can cancel the x in the numerator after the substitution. So, you can evaluate this integral using the "standard" i.e. following the reasoning like on the handout "The Indefinite Integral – Review". Thus we have that

$$\int \frac{x}{\sqrt{1-9x^2}}\, dx = \int \frac{x}{\sqrt{u}} \frac{du}{-18x} = \frac{-1}{18} \int u^{-1/2} du = \frac{-2}{18} u^{1/2} + c = \frac{-1}{9}\sqrt{1-9x^2} + c.$$

(b) This problem is the same as part (a) of Example 3.

(c) Separate the integral into a sum of two. The first matches part (a) and the second part (b). Thus we have that

$$\int \frac{x+1}{\sqrt{1-9x^2}}\, dx = \int \frac{x}{\sqrt{1-9x^2}}\, dx + \int \frac{1}{\sqrt{1-9x^2}}\, dx = -\frac{1}{9}\sqrt{1-9x^2} + \frac{1}{3}\tan^{-1}(3x) + c.$$

Practice Problems:

1. Find all solutions of the following equations on interval $[0, 2\pi]$.

   (a) $\sin x = \frac{2}{5}$ (b) $\cos^2 x = \frac{1}{4}$

   (c) $2\tan x + 3 = 9$ (d) $2\cos^2 x + \cos x - 1 = 0$

   (e) $\sin x \cos x = \sin x$ (f) $\sin x = \cos x$

2. Find the derivatives of the following functions.

   (a) $y = \cot x^2$ (b) $y = x^2 \cos x^2$

   (c) $y = \sin(3x+2)\cos(2x-3)$ (d) $y = \sin^{-1}(2x)$

   (e) $y = \sin(ax)\cos(bx)$ where a and b are arbitrary constants.

   (f) $y = \sin^{-1}(x^4)$ (g) $y = x^2 \cos^{-1} x$

   (h) $y = \tan^{-1}(e^x)$ (i) $y = e^{\tan^{-1} x}$

3. Evaluate the following integrals.

   (a) $\int \cos(3x+1)\, dx$ (b) $\int x \sin x^2\, dx$ (c) $\int (9 + 2\sin\frac{\pi x}{5})\, dx$

   (d) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\, dx$ (e) $\int \frac{1}{\sqrt{1-4x^2}}\, dx$ (f) $\int \frac{1}{4x^2+1}\, dx$

   (g) $\int \frac{1}{x^2+4}\, dx$ (h) $\int \frac{x+1}{x^2+4}\, dx$ (i) $\int \frac{x+3}{x^2+9}\, dx$

(j) (Extra credit level) $\int \frac{x+6}{x^2+4x+13}\,dx$

4. Find the area of the region between $y = \sin x$ and $y = \cos x$ for x in $[0, 2\pi]$.

Solutions.

1. (a) $\sin x = \frac{2}{5} \Rightarrow x = \sin^{-1}\frac{2}{5}$ and $x = \pi - \sin^{-1}\frac{2}{5} \Rightarrow x \approx .411$ and $x \approx 2.73$ radians or 23.57 and 156.42 degrees.

   (b) $\cos^2 x = \frac{1}{4} \Rightarrow \cos x = \pm\frac{1}{2} \Rightarrow x = \pm\cos^{-1}\frac{1}{2}$, $x = \pm\cos^{-1}\frac{-1}{2} \Rightarrow x = \pm\frac{\pi}{3}$, $x = \pm\frac{2\pi}{3}$. Converted to values in the interval $[0, 2\pi]$, the four solutions are $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ and $\frac{5\pi}{3}$ or 60, 120, 240, and 300 degrees.

   (c) $2\tan x + 3 = 9 \Rightarrow 2\tan x = 6 \Rightarrow \tan x = 3 \Rightarrow x = \tan^{-1}(3)$ and $x = \pi + \tan^{-1}(3) \Rightarrow x \approx 1.25$ and $x \approx 4.39$ radians or 71.56 and 251.57 degrees.

   (d) $2\cos^2 x + \cos x - 1 = 0 \Rightarrow (2\cos x - 1)(\cos x + 1) = 0 \Rightarrow \cos x = \frac{1}{2}$, $\cos x = -1 \Rightarrow x = \pm\cos^{-1}\frac{1}{2} = \pm\frac{\pi}{3}$, $x = \pm\cos^{-1}(-1) = \pm\pi$. Converted to values in the interval $[0, 2\pi]$, the solutions are $\frac{\pi}{3}, \pi$, and $\frac{5\pi}{3}$ or 60, 180, and 300 degrees.

   (e) $\sin x \cos x = \sin x \Rightarrow \sin x \cos x - \sin x = \sin x(\cos x - 1) = 0 \Rightarrow \sin x = 0$, $\cos x = 1$. The first equation has solutions 0 and π and the second just 0. Thus 0 and π are the solutions.

   (f) One way to solve this equation is to divide by $\cos x$ so that it reduces to $\tan x = 1$. Thus $x = \tan^{-1}(1) = \frac{\pi}{4}$ and $x = \pi + \tan^{-1}(1) = \frac{5\pi}{4}$. Another way is to convert cos to sine (or vice versa). In this case the equation reduces to $\sin x = \sqrt{1 - \sin^2 x} \Rightarrow \sin^2 x = 1 - \sin^2 x \Rightarrow \sin^2 x = \frac{1}{2} \Rightarrow \sin x = \pm\frac{1}{\sqrt{2}}$. Be careful about the extraneous roots $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$.

2. (a) Representing the function as $y = \cot x^2 = \frac{\cos x^2}{\sin x^2}$ and using the quotient rule with $f(x) = \cos x^2$ and $g(x) = \sin x^2$ and the chain for $f'(x) = -\sin x^2(2x)$ and $g'(x) = \cos x^2(2x)$ obtain that $y' = \frac{-\sin x^2(2x)\sin x^2 - \cos x^2(2x)\cos x^2}{\sin^2 x^2} = \frac{-2x(\sin^2 x^2 + \cos^2 x^2)}{\sin^2 x^2} = \frac{-2x}{\sin^2 x^2}$.

   (b) The product rule with $f(x) = x^2$ and $g(x) = \cos x^2$ and the chain for $g'(x) = -\sin x^2(2x)$ so that $y' = 2x\cos x^2 - \sin x^2(2x)(x^2) = 2x\cos x^2 - 2x^3\sin x^2$.

   (c) Product and chain: $y' = 3\cos(3x+2)\cos(2x-3) - 2\sin(2x-3)\sin(3x+2)$.

   (d) Use the chain rule with the outer $\sin^{-1}(u)$ and the inner $2x$. The derivative of the outer with the inner unchanged is $\frac{1}{\sqrt{1-(2x)^2}}$ and the derivative of the inner is 2. Thus $y' = \frac{2}{\sqrt{1-4x^2}}$.

   (e) Product and chain: $y' = a\cos(ax)\cos(bx) - b\sin(bx)\sin(ax)$.

   (f) Use the chain rule with the outer $\sin^{-1}(u)$ and the inner x^4. The derivative of the outer with the inner unchanged is $\frac{1}{\sqrt{1-(x^4)^2}}$ and the derivative of the inner is $4x^3$. Thus $y' = \frac{4x^3}{\sqrt{1-x^8}}$.

   (g) Use the product rule. $y' = 2x\cos^{-1} x - \frac{x^2}{\sqrt{1-x^2}}$.

   (h) Use the chain rule with the outer $\tan^{-1}(u)$ and the inner e^x. The derivative of the outer with the inner unchanged is $\frac{1}{1+(e^x)^2}$ and the derivative of the inner is e^x. Thus $y' = \frac{e^x}{1+e^{2x}}$.

   (i) Use the chain rule with the outer e^u and the inner $\tan^{-1} x$. The derivative of the outer with the inner unchanged is $e^u = e^{\tan^{-1} x}$ and the derivative of the inner is $\frac{1}{1+x^2}$. Thus $y' = \frac{e^{\tan^{-1} x}}{1+x^2}$.

3. (a) Use the substitution $u = 3x + 1$. The antiderivative is $\frac{1}{3}\sin(3x+1) + c$.

(b) Use the substitution $u = x^2$. The antiderivative is $\frac{-1}{2}\cos(x^2) + c$.

(c) Use the substitution $u = \frac{\pi x}{5} \Rightarrow du = \frac{\pi}{5}dx \Rightarrow \frac{5du}{\pi} = dx$. The integral becomes $\int(9 + 2\sin u)\frac{5du}{\pi} = \frac{5}{\pi}\int(9+2\sin u)du = \frac{5}{\pi}(9u - 2\cos u) + c = \frac{5}{\pi}(9\frac{\pi x}{5} - 2\cos\frac{\pi x}{5}) + c = 9x - \frac{10}{\pi}\cos\frac{\pi x}{5} + c$.

(d) The antiderivative is $\sin^{-1} x$. Substituting the bounds, you obtain $\sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{2}$.

(e) Note that the integral function has the form $\frac{1}{\sqrt{1-u^2}}$ that yields $\sin^{-1} u$ with $4x^2$ being u^2. This tells you that $4x^2 = u^2$ (*careful:* not u but u^2). With $u^2 = 4x^2$, you can have $u = 2x$. Thus, $du = 2dx \Rightarrow \frac{du}{2} = dx$. Substitute and obtain $\int \frac{1}{\sqrt{1-u^2}}\frac{du}{2} = \frac{1}{2}\sin^{-1} u + c = \frac{1}{2}\sin^{-1}(2x) + c$.

(f) Note that the integral function has the form $\frac{1}{1+u^2}$ that yields $\tan^{-1} x$ with $4x^2$ being u^2. This tells you that $4x^2 = u^2$. So, you can take $u = 2x$. Thus, $du = 2dx \Rightarrow \frac{du}{2} = dx$. Substitute and obtain $\int \frac{1}{u^2+1}\frac{du}{2} = \frac{1}{2}\tan^{-1} u + c = \frac{1}{2}\tan^{-1}(2x) + c$.

(g) To obtain the form $\frac{1}{1+u^2}$, factor 4 out of the denominator first. $\int \frac{1}{x^2+4}\,dx = \int \frac{1}{4(\frac{x^2}{4}+1)}dx = \frac{1}{4}\int \frac{1}{\frac{x^2}{4}+1}dx$. Note that the integral function is of the form $\frac{1}{1+u^2}$, with $u^2 = \frac{x^2}{4}$. So, you can take $u = \frac{x}{2}$. Thus, $du = \frac{dx}{2} \Rightarrow 2du = dx$. Substitute and obtain $\frac{1}{4}\int \frac{1}{u^2+1}\,2du = \frac{1}{2}\tan^{-1} u + c = \frac{1}{2}\tan^{-1}(\frac{x}{2}) + c$.

(h) Note that the function $\frac{x+1}{x^2+4}$ is the sum $\frac{x}{x^2+4} + \frac{1}{x^2+4}$. Integrate both terms. The first integral can be evaluated using the substitution $u = x^2 + 4 \Rightarrow du = 2xdx \Rightarrow \frac{du}{2x} = dx$. Thus, $\int \frac{x}{x^2+4}dx = \int \frac{x}{u}\frac{du}{2x} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln(x^2+4)$. The second integral reduces to the previous problem with substitution $u = \frac{x}{2}$ and solution $\frac{1}{2}\tan^{-1}(\frac{x}{2})$. Thus, the final answer is $\frac{1}{2}\ln(x^2+4) + \frac{1}{2}\tan^{-1}(\frac{x}{2}) + c$.

(i) Note that the function $\frac{x+3}{x^2+9}$ is the sum $\frac{x}{x^2+9} + \frac{3}{x^2+9}$. Integrate both terms. The first integral can be evaluated using the substitution $u = x^2 + 9 \Rightarrow du = 2xdx \Rightarrow \frac{du}{2x} = dx$. Thus, $\int \frac{x}{x^2+9}dx = \int \frac{x}{u}\frac{du}{2x} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln(x^2+9)$.

The second integral is similar to problem 7. To obtain the form $\frac{1}{1+u^2}$, factor 9 out of the denominator first. You can also factor 3 out so that it is out of your way. $\int \frac{3}{x^2+9}\,dx = \frac{3}{9}\int \frac{1}{\frac{x^2}{9}+1}dx$. Note that the integral function is of the form $\frac{1}{1+u^2}$, with $u^2 = \frac{x^2}{9}$. So, you can take $u = \frac{x}{3}$. Thus, $du = \frac{dx}{3} \Rightarrow 3du = dx$. Substitute and obtain $\frac{1}{3}\int \frac{1}{u^2+1}\,3du = \tan^{-1} u + c = \tan^{-1}(\frac{x}{3}) + c$. Thus, the final answer is $\frac{1}{2}\ln(x^2+9) + \tan^{-1}(\frac{x}{3}) + c$.

(j) To apply the ideas for solving previous problems to this one, you want to complete the denominator to squares first (that is: to write the quadratic of the form $ax^2 + bx + c$ in the form $(px+q)^2 + r^2$.). The denominator $x^2 + 4x + 13$ is equal to $x^2 + 2(2)x + 2^2 + 9$. Note that the first three terms are equal to $(x+2)^2$. Thus, the denominator is equal to $(x+2)^2 + 9$.

This tells you that you want to evaluate the integral $\int \frac{x+6}{x^2+4x+13}\,dx$ as the sum of two integrals. For the first you will take the whole denominator $(x+2)^2+9$ for u. This indicates how to decompose the numerator so that $du = 2(x+2)dx \Rightarrow \frac{du}{2(x+2)} = dx$ cancels the x-terms in the numerator. Thus $\int \frac{x+6}{x^2+4x+13}\,dx = \int \frac{x+2+4}{(x+2)^2+9}dx = \int \frac{x+2}{(x+2)^2+9}dx + \int \frac{4}{(x+2)^2+9}dx$.

For the first integral you obtain $\int \frac{1}{u}\frac{du}{2} = \frac{1}{2}\ln u = \frac{1}{2}\ln((x+2)^2+9)$.
Reduce the second integral to formula $\frac{1}{u^2+1}$ that will yield $\tan^{-1}(u)$. Note that the denominator $(x+2)^2+9$ is equal to $9[(\frac{x+2}{3})^2+1]$. So, for the second integral, you can use the substitution $u = \frac{x+2}{3} \Rightarrow du = \frac{dx}{3} \Rightarrow 3du = dx$. This integral becomes $= \frac{4}{9}\int \frac{1}{(\frac{x+2}{3})^2+1}dx = \frac{4}{9}3\int \frac{1}{u^2+1}du = \frac{4}{3}\tan^{-1}u = \frac{4}{3}\tan^{-1}\frac{x+2}{3}$. Thus, the final answer is $\frac{1}{2}\ln((x+2)^2+9) + \frac{4}{3}\tan^{-1}\frac{x+2}{3} + c$.

4. Find intersections. The equation $\sin x = \cos x$ has solutions $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ by problem 1 (f). Graph the functions and note that the area consists of 3 regions as in the graph on the right. $A = A_1 + A_2 + A_3 = \int_0^{\pi/4}(\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)dx + \int_{5\pi/4}^{2\pi}(\cos x - \sin x)dx = (\sin x + \cos x)|_0^{\pi/4} + (-\cos x - \sin x)|_{\pi/4}^{5\pi/4} + (\sin x + \cos x)|_{5\pi/4}^{2\pi} = 4\sqrt{2} \approx 5.657$.