

Formulas for Exam 1

1. Derivatives.

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|------|------------|-------|-------------|---------------|-------------------------------------|----------|-----------|--------------------------|-------------------|---------------------------|
| y | x^n | e^x | b^x | $\ln x$ | $\log_b x$ | $\sin x$ | $\cos x$ | $\sin^{-1} x$ | $\tan^{-1} x$ | $\sec^{-1} x$ |
| y' | nx^{n-1} | e^x | $b^x \ln b$ | $\frac{1}{x}$ | $\frac{1}{x} \cdot \frac{1}{\ln b}$ | $\cos x$ | $-\sin x$ | $\frac{1}{\sqrt{1-x^2}}$ | $\frac{1}{1+x^2}$ | $\frac{1}{x\sqrt{x^2-1}}$ |

2. Integrals.

| | | | | | | | | |
|-------------|------------------------|-------|-----------------------|---------------|-----------|----------|--------------------------|-------------------|
| y | x^n | e^x | b^x | $\frac{1}{x}$ | $\sin x$ | $\cos x$ | $\frac{1}{\sqrt{1-x^2}}$ | $\frac{1}{1+x^2}$ |
| $\int y dx$ | $\frac{1}{n+1}x^{n+1}$ | e^x | $\frac{1}{\ln b} b^x$ | $\ln x $ | $-\cos x$ | $\sin x$ | $\sin^{-1} x$ | $\tan^{-1} x$ |

3. Rules of Differentiation.

- a) Product rule: If $y = f \cdot g$, then $y' = f' \cdot g + g' \cdot f$
- b) Quotient rule: If $y = \frac{f}{g}$, then $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$
- c) Chain rule: If $y = f(g(x))$, then $y' = f'(g(x)) \cdot g'(x)$

4. Integration by parts. $\int u dv = uv - \int v du$

5. Vectors. If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, then

- a) The **length** of \vec{a} is $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.
- b) The **normalized vector** of \vec{a} is $\frac{\vec{a}}{|\vec{a}|}$.
- c) The **dot product** is $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.
- d) The **projection** of \vec{a} onto \vec{b} is $\text{proj}_{\vec{b}} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$.
- e) The **cross product** is $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

f) Given the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the vector \vec{PQ} is

$$\vec{PQ} = \vec{OQ} - \vec{OP} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle.$$

6. Line through the point (x_0, y_0, z_0) in the direction of $\langle a, b, c \rangle$ is:

$$x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$$

7. Plane through the point (x_0, y_0, z_0) perpendicular to $\langle a, b, c \rangle$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

Simplifies to $ax + by + cz = d$.

8. Curve $x = x(t)$ $y = y(t)$ $z = z(t)$.

The **line tangent** to this curve at the point where $t = t_0$ passes $(x(t_0), y(t_0), z(t_0))$ in the direction of $\langle x'(t_0), y'(t_0), z'(t_0) \rangle$. So, it has equations

$$x = x(t_0) + x'(t_0)t \quad y = y(t_0) + y'(t_0)t \quad z = z(t_0) + z'(t_0)t$$

The **length** of this curve from point where $t = a$ to point where $t = b$ is

$$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} dt = \int_a^b |\vec{r}'(t)| dt$$

9. Surfaces.

- a) A vector perpendicular to the **tangent plane** to the surface $z = f(x, y)$ at the point (x_0, y_0, z_0) is

$$\langle f_x(x_0, y_0), f_y(x_0, y_0), -1 \rangle.$$

- b) If a surface is given implicitly as $F(x, y, z) = 0$. Then partial derivatives z_x and z_y are given by:

$$z_x = -\frac{F_x}{F_z} \quad \text{and} \quad z_y = -\frac{F_y}{F_z}$$

- c) A vector perpendicular to the **tangent plane** to the surface $F(x, y, z) = 0$ at the point (x_0, y_0, z_0) is

$$\langle F_x(x_0, y_0, z_0), F_y(x_0, y_0, z_0), F_z(x_0, y_0, z_0) \rangle.$$

- d) The **linear approximation** of $z = f(x, y)$ with a tangent plane to $z_0 = f(x_0, y_0)$ is

$$f(x, y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

10. **Chain Rule.** Derivative of the function $z = f(x, y)$ with $x = g(t)$ and $y = h(t)$ is

$$z'(t) = z_x \cdot x'(t) + z_y \cdot y'(t)$$

The partial derivatives of the function $z = f(x, y)$ with $x = g(s, t)$ and $y = h(s, t)$ are

$$z_s = z_x \cdot x_s + z_y \cdot y_s \qquad z_t = z_x \cdot x_t + z_y \cdot y_t$$