Calculus 3
Lia Vas

Formulas for Exam 1

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \lvert x\rvert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Rules of Differentiation.**

   a) Product rule: If $y = f \cdot g$, then $y' = f' \cdot g + g' \cdot f$

   b) Quotient rule: If $y = \frac{f}{g}$, then $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$

   c) Chain rule: If $y = f(g(x))$, then $y' = f'(g(x)) \cdot g'(x)$

4. **Integration by parts.** $\int u\ dv = u\, v - \int v\, du$

5. **Finding the second solution of some basic trigonometric equations.**

$$\sin x = a \quad \Rightarrow \quad x_1 = \sin^{-1}(a) \quad \text{and} \quad x_2 = \pi - \sin^{-1}(a)$$
$$\cos x = a \quad \Rightarrow \quad x_1 = \cos^{-1}(a) \quad \text{and} \quad x_2 = -\cos^{-1}(a)$$
$$\tan x = a \quad \Rightarrow \quad x_1 = \tan^{-1}(a) \quad \text{and} \quad x_2 = \pi + \tan^{-1}(a)$$

6. **Vectors.** If $\vec{a} = \langle a_1, a_2, a_3\rangle$ and $\vec{b} = \langle b_1, b_2, b_3\rangle$, then

   a) The **length** of $\vec{a}$ is $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$. b) The **normalized vector** of $\vec{a}$ is $\frac{\vec{a}}{|\vec{a}|}$.

   c) The **dot product** is $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$. d) The projection of $\vec{a}$ onto $\vec{b}$ is $\text{proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\vec{b}$. e) The **cross product** is $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

   f) Given the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the vector $\overrightarrow{PQ}$ is

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1\rangle.$$

7. **Line** through the point (x_0, y_0, z_0) in the direction of $\langle a, b, c\rangle$ is:

$$x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$$

8. **Plane** through the point (x_0, y_0, z_0) perpendicular to $\langle a, b, c\rangle$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

   Simplifies to $ax + by + cz = d$.

9. **Curve** $x = x(t) \quad y = y(t) \quad z = z(t)$.

   The **line tangent** to this curve at the point where $t = t_0$ passes $(x(t_0), y(t_0), z(t_0))$ in the direction of $\langle x'(t_0), y'(t_0), z'(t_0)\rangle$. So, it has equations

   $$x = x(t_0) + x'(t_0)t \quad y = y(t_0) + y'(t_0)t \quad z = z(t_0) + z'(t_0)t$$

   The **length** of this curve from point where $t = a$ to point where $t = b$ is

   $$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2 dt} = \int_a^b |\overrightarrow{r'(t)}| dt$$

10. **Surfaces.**

    a) A vector perpendicular to the **tangent plane** to the surface $z = f(x, y)$ at the point (x_0, y_0, z_0) is

    $$\langle z_x, z_y, -1\rangle$$

    evaluated at the point (x_0, y_0, z_0).

    b) If a surface is given implicitly as $F(x, y, z) = 0$. Then partial derivatives z_x and z_y are given by:

    $$z_x = -\frac{F_x}{F_z} \quad \text{and} \quad z_y = -\frac{F_y}{F_z}$$

    c) A vector perpendicular to the **tangent plane** to the surface $F(x, y, z) = 0$ at the point (x_0, y_0, z_0) is

    $$\langle F_x, F_y, F_z\rangle$$

    evaluated at (x_0, y_0, z_0).

    d) The **linear approximation** of $z = f(x, y)$ with a tangent plane to $z_0 = f(x_0, y_0)$ is

    $$f(x, y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

11. **Chain Rule.** Derivative of the function $z = z(x, y)$ with $x = x(t)$ and $y = y(t)$ is

    $$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} \quad \text{equivalentlly,} \quad z'(t) = z_x\, x'(t) + z_y\, y'(t)$$

    The partial derivatives of the function $z = z(x, y)$ with $x = x(s, t)$ and $y = y(s, t)$ are

    $$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

    equivalentlly,

    $$z_s = z_x\, x_s + z_y\, y_s \qquad z_t = z_x\, x_t + z_y\, y_t.$$