

Matlab Notes for Calculus 3

Lia Vas

Content

0. Review of Matlab.
Representing Functions. Solving Equations.
Basic Graphing. Differentiation and Integration.
1. Vectors.
2. Differentiation of Multi-variable Functions (Practice problems 1)
3. Graphing Curves, Surfaces, and Vector Fields
 - 3.1 Curves
 - 3.2 Surfaces
 - 3.3 Vector Fields
 - 3.4 Special effects (Practice problems 2)
4. Integration of Multi-variable Functions (Practice problems 3)
5. Series and Taylor Series (Practice problems 4)

0. Review of Matlab

Representing Functions. Solving Equations.

You can use +, -, *, \ and ^ to add, subtract, multiply, divide or exponentiate, respectively. To perform symbolic calculations, use **syms** to declare the variables you plan to use. For example, suppose that you need factor x^2-3x+2 . First you need

syms x (you are declaring that x is a variable)

Then you can use the command **factor(x^2-3*x+2)** to get the answer **ans =(x-1)*(x-2)**

Note that we entered **3*x** to represent $3x$ in the command above. **Entering * for multiplication is always necessary in Matlab.**

For solving equations, you can use the command **solve**. This command has the following form:

solve('equation', 'variable for which you are solving')

For example, to solve the equation $3x^2-8x+2=0$ you can use

solve('3*x^2-8*x+2=0','x')

and obtain **ans =** $\left[\frac{4}{3} + \frac{1}{3} \sqrt{10} \right]$ $\left[\frac{4}{3} - \frac{1}{3} \sqrt{10} \right]$

If we want to get the answer in the decimal form with, say, three significant digits, we can use the command **vpa**.

vpa(ans, 3) **ans = [2.38] [0.28]**

By changing 3 in the command **vpa(ans, 3)** you can specify the number of digits in the answer.

You can solve an equation in two variables for one of them. For example the command

solve('y^2-5*x*y-y+6*x^2+x=2', 'y')

solves the given equation for values of y in terms of x. The answer is:

ans = [3*x+2] [2*x-1]

You can solve more than one equation simultaneously. For example suppose that we need to solve the system $x^2 + x + y^2 = 2$ and $2x - y = 2$. We can use:

[x,y]=solve('x^2+ x+ y^2 = 2', '2*x-y = 2')

to get the answer

x = [2/5] [1]

y = [-6/5] [0]

Note that the **[x,y]=** part at the beginning of the command was necessary since without it you obtain the answer:

ans = x: [2x1 sym] y: [2x1 sym]

This answer tells us just that the solutions are two values of the pair (x,y) but we do not get the solutions themselves. To get the solution vectors displayed, we must use **[x,y]=** before the command **solve**.

The following table gives an overview of the way most commonly used functions or expressions are represented in Matlab.

function or symbol	representation in MATLAB
e^x	exp(x)
$\ln x$	log(x)
$\log x$	log(x)/log(10)
log. base a of x	log(x)/log(a)
$\sin x$	sin(x)
$\cos x$	cos(x)
arctan(x)	atan(x)
π	pi

To represent a function, use the command **inline**. This command has the following form:

inline('function', 'independent variable of the function')

To define the function x^2+3x-2 use **f = inline('x^2+3*x-2', 'x')**

f = Inline function: f(x) = x^2+3*x-2

After defining a function, we can evaluate it at a point. For example, the command **f(2)** gives you the answer **ans = 8**

Just as when using calculator, one must be careful when representing a function. For example

$\frac{1}{x(x+6)}$ should be represented as **1/(x*(x+6))** not as **1/x*(x+6)** nor as **1/x(x+6)**,

$\frac{3}{x^2+5x+6}$ should be represented as **3/(x^2+5*x+6)** not as **3/x^2+5*x+6**,

e^{5x^2} should be represented as **exp(5*x^2)** not as $e^{(5*x^2)}$, $\exp^*(5*x^2)$, $\exp(5x^2)$ nor as $\exp^{\wedge}(5*x^2)$.

Basic Graphing

When graphing functions depending on a variable x , you can start your session by declaring x for your variable by **syms x**

The simplest way to graph a function is to use the command **ezplot** (easy plot). For example, to graph the function x^2+x+1 , use **ezplot(x^2+x+1)**

If we want the variable x to take values between -2 and 2, use **ezplot(x^2+x+1, [-2, 2])**

If we want to see the graph for x in the interval [-2, 2] and for y in the interval [1, 4], use **axis([-2 2 1 4])**

To plot multiple curves on the same window, you can also use the **ezplot** command. For example: **ezplot('sin(x)') hold on ezplot('exp(-x^2)') hold off**

Differentiation and Integration

The command for differentiation is **diff**. It has the following form **diff(function)**

For example, to find derivative of x^3-2x+5 use **diff('x^3-2*x+5')** or **syms x** and **diff(x^3-2*x+5)** gives you the answer **ans = 3*x^2-2**

For the n -th derivative use **diff(function, n)**

For example, to get the second derivative of x^3-2x+5 , use **diff(x^3-2*x+5, 2)**
ans = 6*x

Similarly, the 23rd derivative of $\sin(x)$ is obtained by **diff(sin(x), 23)**
ans = -cos(x)

For the indefinite integrals, start with **syms x** followed by the command **int(function)**

For example, the command **int(x^2)** evaluates the integral $\int x^2 dx$ and gives us the answer **ans = 1/3*x^3**

For definitive integrals, the command is **int(function, lower bound, upper bound)**

For example, **int(x^2, 0, 1)** evaluates the integral $\int_0^1 x^2 dx$ The answer is **ans = 1/3**

1. Vectors

You can enter a vector by typing a list of numbers separated by either commas or spaces inside the square brackets. For example, you can represent vectors (1,2,3) and (4,5,6) by

X = [1,2,3]

Y = [4 5 6]

You can add vectors and multiply them with numbers as follows.

X+Y (addition) **ans = 5 7 9**

3*X (scalar multiplication) **ans = 3 6 9**

The commands **dot** and **cross** compute the dot and cross product of vectors.

For example, with **X** and **Y** as above, the dot product is computed by

dot(X,Y) and produces the answer

ans = 32

The cross product can be computed by

cross(X,Y) producing the answer

ans = -3 6 -3

The command **norm(X)** computes the length of a vector **X**. For example, if **X = [1,2,3]**

norm(X) computes the length to be

ans = 3.7417

2. Differentiation of Multi-variable Functions

The command **diff** can be used to compute **partial derivatives**. For example, to find the first partial derivatives z_x and z_y of the function $z=x^2y^3$, you can first use

syms x y

to represent x and y as variables followed by

diff(x^2*y^3, x) **ans = 2*x*y^3**

to compute z_x and

diff(x^2*y^3, y) **ans = 3*x^2*y^2**

to compute z_y .

To find the second partial derivative z_{xx} with respect to x , you can use

diff(x^2*y^3, x, 2) **ans = 2*y^3**

And similarly you can find z_{yy} .

For the mixed second partial derivative z_{xy} (in this case equal to z_{yx}) you can use the following.

diff(diff(x^2*y^3, x), y) **ans = 6*x*y^2**

Similarly, the following command computes the derivative of $z = 3x^9y^7$ twice with respect to x and three times with respect to y .

diff(diff(3*x^9*y^7, x, 2), y, 3)

Practice problems 1

1. Determine if vectors $\mathbf{X} = [1, -3, 4]$ and $\mathbf{Y} = [2, 5, 3]$ are perpendicular, parallel or neither. Also find the length of vector \mathbf{X} .
2. Find the first partial derivatives of the function $\frac{\sin xy}{\ln(x^2+1)} - e^y$.
3. Find all of the first and second partial derivatives of the function $e^{x^2} \sin xy$.

Solutions.

1. Enter the vectors as $\mathbf{X} = [1, -3, 4]$ and $\mathbf{Y} = [2, 5, 3]$. Then compute the dot and cross products. **dot(X,Y) ans = -1**. Since the dot product is not zero, the vectors are not perpendicular. **cross(X,Y) ans = -29 5 11**. Since the cross product is not zero, the vectors are not parallel. So, they are neither perpendicular nor parallel. The length of \mathbf{X} can be found by **norm(X) ans = sqrt(26) or 5.099**.
2. If we denote the given function by z , the derivatives z_x and z_y can be found as follows. **diff(sin(x*y)/log(x^2+1)-exp(y), x)** and **diff(sin(x*y)/log(x^2+1)-exp(y), y)**.
3. For the given function z , obtain z_x , z_y , z_{xx} , z_{xy} , and z_{yy} as follows.
diff(exp(x^2)*sin(x*y), x), diff(exp(x^2)*sin(x*y), y),
diff(exp(x^2)*sin(x*y), x, 2), diff(diff(exp(x^2)*sin(x*y), x), y), and
diff(exp(x^2)*sin(x*y), y, 2).

3. Graphing Curves, Surfaces, and Vector Fields

3.1 Curves

Recall that you can use the command **ezplot** to graph a parametric curve in xy-plane by **ezplot('x-coordinate', 'y-coordinate', [parameter domain])**

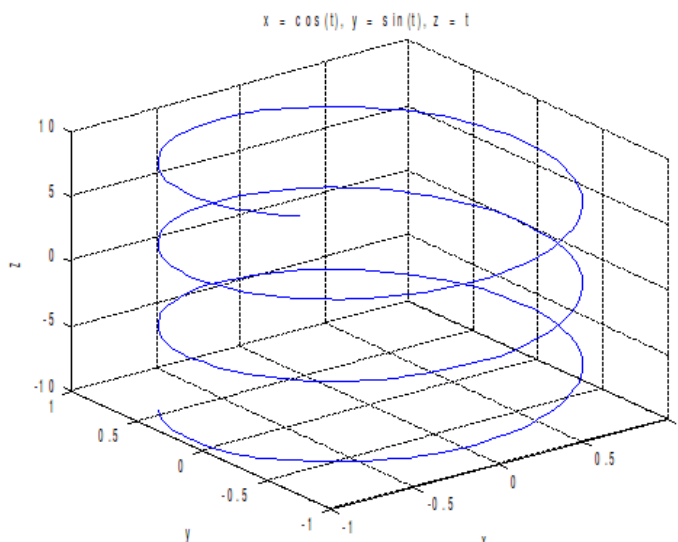
For example, to graph a circle $x = \cos t$, $y = \sin t$ for $0 \leq t \leq 2\pi$, you can use:
ezplot('cos(t)', 'sin(t)', [0, 2*pi])

This command translates to three-dimensional space as follows.

ezplot3('x-coordinate', 'y-coordinate', 'z-coordinate', [parameter domain])

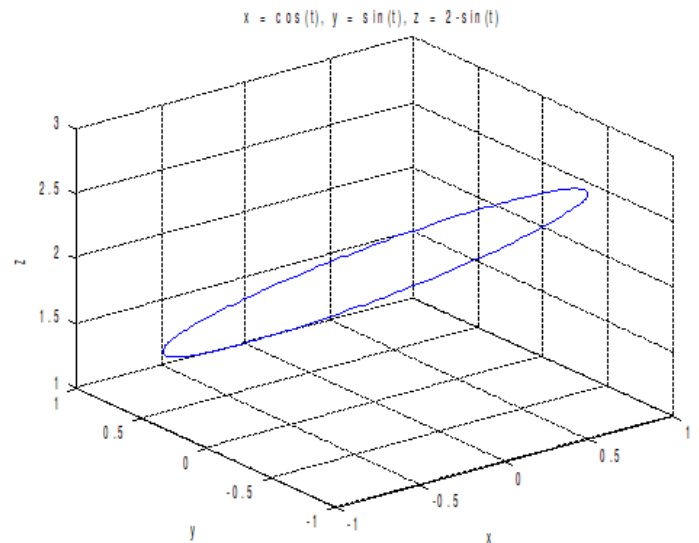
For example, graph the helix $x = \cos t$, $y = \sin t$, $z = t$, for $-10 \leq t \leq 10$ we can use the following command and obtain the graph displayed below.

ezplot3('cos(t)', 'sin(t)', 't', [-10, 10])



In some cases, you may need to parametrize the curve first. For example, to graph the curve in the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$, you first need to find a parametrization of this curve. Since x and y coordinates are related by $x^2 + y^2 = 1$, you can use $x = \cos t$ and $y = \sin t$. From the plane equation $z = 2 - y$ and so $z = 2 - \sin t$. So the curve can be parametrized by $x = \cos t$, $y = \sin t$ and $z = 2 - \sin t$. To get the graph on the right, you can use

```
ezplot3('cos(t)', 'sin(t)', '2-sin(t)', [0, 2*pi])
```



3.2 Surfaces

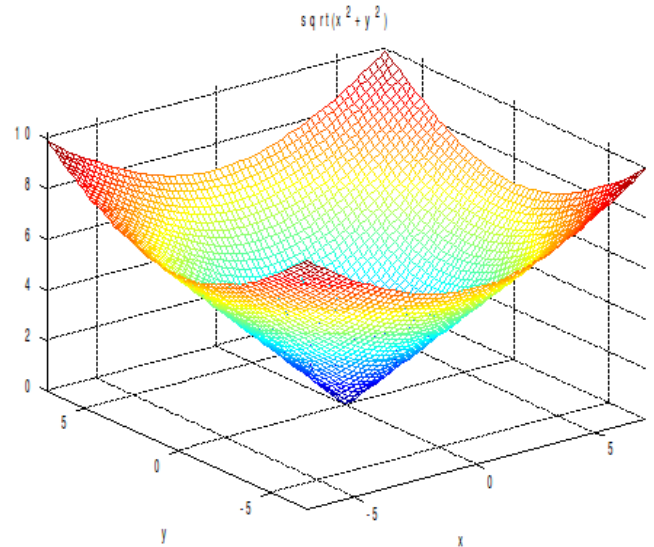
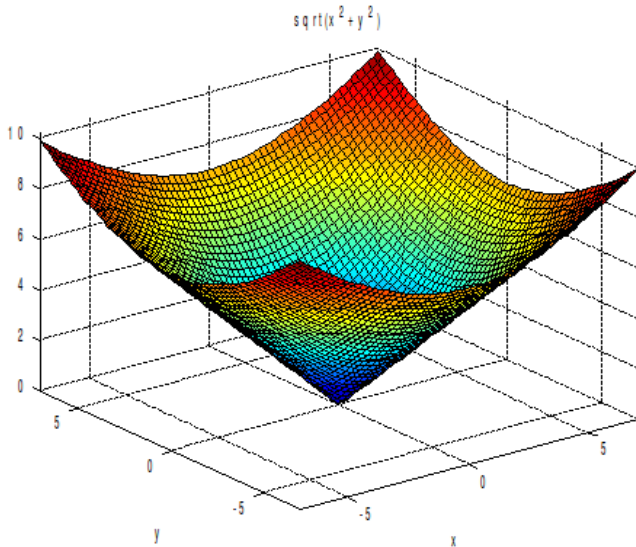
There are two commands for graphing surfaces: **ezmesh** and **ezsurf**. The first produces a transparent wired plot, a mesh surface. The second produces a shaded, nontransparent surface. Both commands can be used for graphing surfaces that are given in the form

$$z = f(x, y)$$

as follows

```
ezsurf( function f, [x-domain, y-domain])
ezmesh( function f, [x-domain, y-domain])
```

For example, the graph of the cone $z = \sqrt{x^2 + y^2}$ over the square $-7 \leq x \leq 7$, $-7 \leq y \leq 7$, can be obtained using **ezsurf** command by **ezsurf('sqrt(x^2+y^2)', [-7, 7], [-7, 7])** and using **ezmesh** command by **ezmesh('sqrt(x^2+y^2)', [-7, 7], [-7, 7])**



Parametric Surfaces. The commands **ezmesh** and **ezsurf** can also be used for graphing the surfaces given in the parametric form

$$x = x(u, v), \quad y = y(u, v), \quad z = z(u, v).$$

In this case, the commands **ezsurf** and **ezmesh** have the following forms

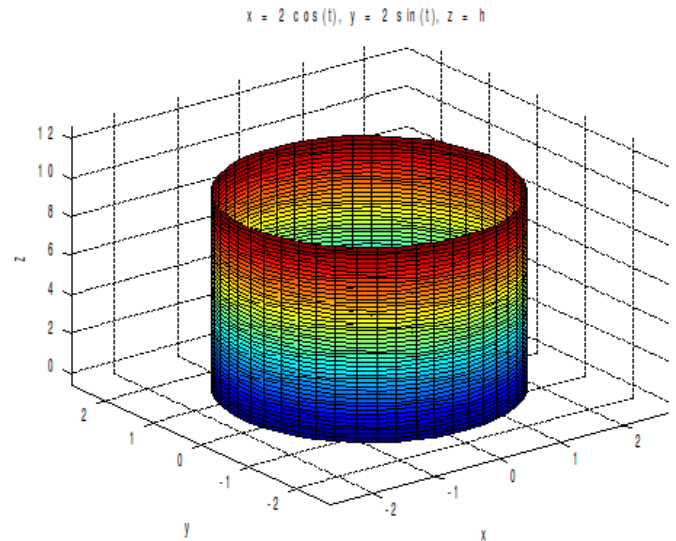
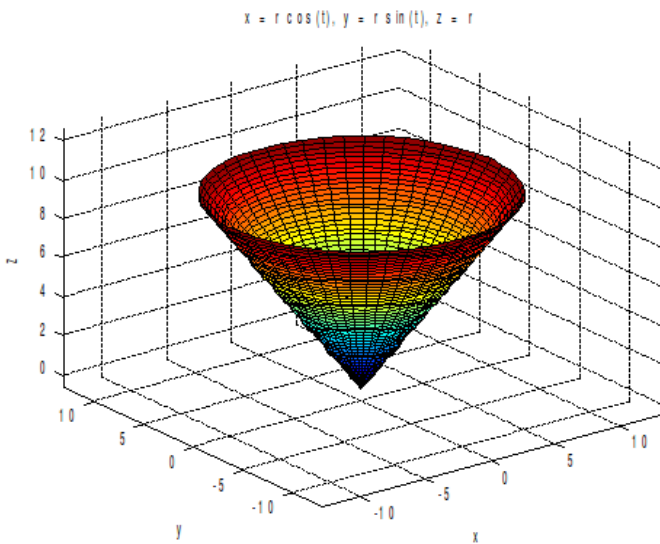
ezsurf('x-coordinate', 'y-coordinate', 'z-coordinate', [u-domain, v-domain])
ezmesh('x-coordinate', 'y-coordinate', 'z-coordinate', [u-domain, v-domain])

Note that the order of the variables in the brackets at the end of the command is alphabetical. For example, the cone $z = \sqrt{x^2 + y^2}$ from the previous example can be represented by parametric equations as $x = r \cos t$, $y = r \sin t$ and $z = r$

To represent it for the values $0 \leq r \leq 10$, for example, we can use

ezsurf('r*cos(t)', 'r*sin(t)', 'r', [0, 10, 0, 2*pi])

or similarly by using **ezmesh** command.



Graphing a cylinder requires the use of parametric equations since its defining relation cannot be written in the form $z=f(x,y)$. For example, to graph the cylinder $x^2 + y^2 = 4$ note that you can parametrize it by

$$x = 2 \cos t, y = 2 \sin t \text{ and } z = h$$

(here the name of the second parameter h was chosen to reflect that it represents the height h). To graph it for the height values $0 \leq h \leq 10$, for example, you can use

ezsurf('2*cos(t)', '2*sin(t)', 'h', [0, 10, 0, 2*pi])

and obtain the second graph above.

Similarly, graphing a sphere requires the use of parametric equations. Recall that parametric equations of a sphere centered at the origin can be obtained by using the spherical coordinates

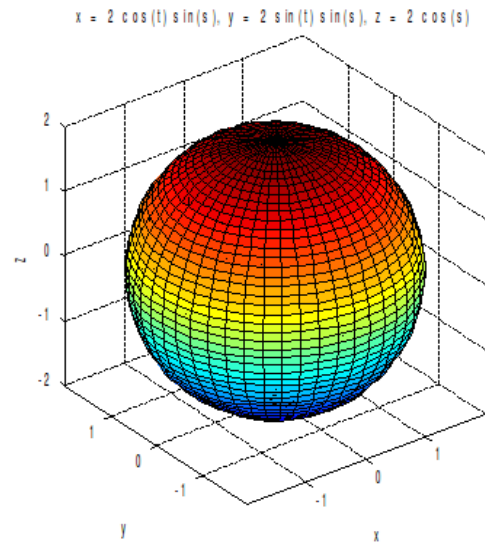
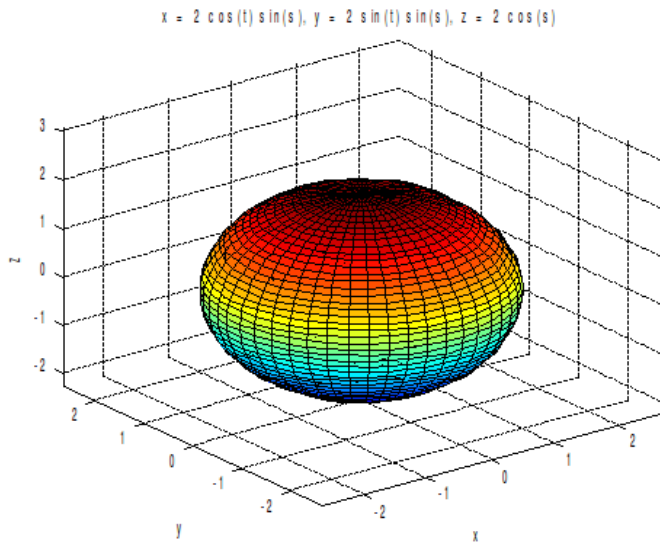
$$x = r \cos \theta \sin \varphi \quad y = r \sin \theta \sin \varphi \quad z = r \cos \varphi$$

with r values being constant and equal to the radius of the sphere. For example, to graph the sphere centered at the origin of radius 2, note that we can parametrize it as

$$x = 2 \cos \theta \sin \varphi \quad y = 2 \sin \theta \sin \varphi \quad z = 2 \cos \varphi.$$

For simplicity, let t denotes the variable θ and s denotes the variable φ . The sphere can be graphed by

`ezsurf('2*cos(t)*sin(s)', '2*sin(t)*sin(s)', '2*cos(s)', [0, pi, 0, 2*pi])`



With the same command followed by `axis equal`, we obtain the second graph below which is more “regular”.

You can graph two surfaces on the same plot by using `hold on` and `hold off` commands. For example, to graph the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$, on the same plot, note that the cylinder can be parametrized by $x = \cos t$, $y = \sin t$ and $z = h$ and the plane by $x=x$, $y=y$ and $z=2-y$.

Thus, you can graph both surfaces by `ezsurf('cos(t)', 'sin(t)', 'h', [0, 5, 0, 2*pi])`

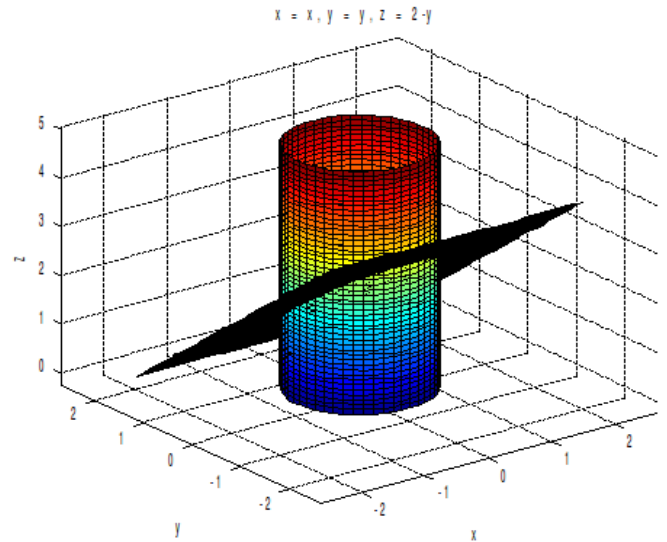
`hold on`

`ezsurf('x','y','2-y', [-2, 2, -2, 2])`

`hold off`

The curve in the intersection is the ellipse (see page 5) which we graphed before by `ezplot3('cos(t)', 'sin(t)', '2-sin(t)', [0, 2*pi])`.

`ezplot3('cos(t)', 'sin(t)', '2-sin(t)', [0, 2*pi])`.



3.3 Vector Fields

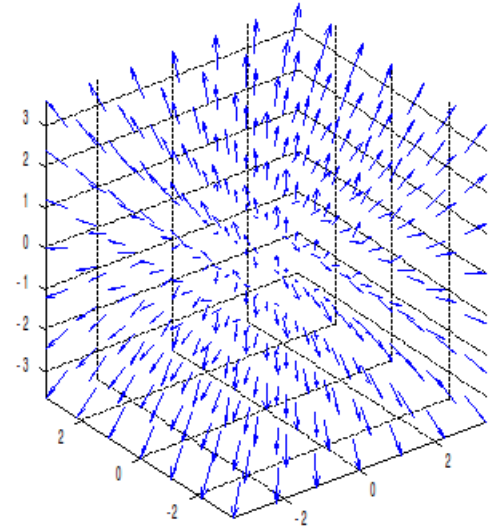
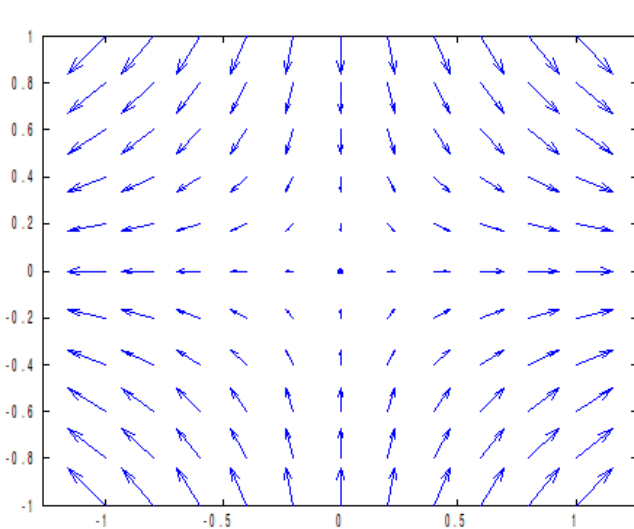
Plotting the two-dimensional vector field $\mathbf{F}(x, y) = P(x, y) \mathbf{i} + Q(x, y) \mathbf{j}$ requires the use of commands `meshgrid` and `quiver`. The command `meshgrid` is used for representing the base of each vector. For example, if we are to graph the vectors starting at points (x,y) for x and y values between -1 and 1 with step size 0.2 from each other, the command

`[x,y]=meshgrid(-1:0.2:1, -1:0.2:1);`

represents these points in Matlab. The command ends with the semicolon since these points need not to be displayed. The command `meshgrid` is followed by the command

quiver(x, y, x-coordinate of the field F, y-coordinate of the field F)

For example, to plot the vector field $\mathbf{F}(x, y) =$



$x \mathbf{i} - y \mathbf{j}$ for $-1 \leq x, y \leq 1$, we can use the following. **[x,y]=meshgrid(-1:.2:1, -1:.2:1);**
quiver(x, y, x, -y) axis equal

The outcome is the first graph below.

To graph the three-dimensional vector field $\mathbf{F}(x, y, z) = P(x, y, z) \mathbf{i} + Q(x, y, z) \mathbf{j} + R(x, y, z) \mathbf{k}$ on a given domain, you can use the command **meshgrid** followed by the command **quiver3** which, in this case, has the following format

quiver3(x, y, z, x-coordinate of F, y-coordinate of F, z-coordinate of F)

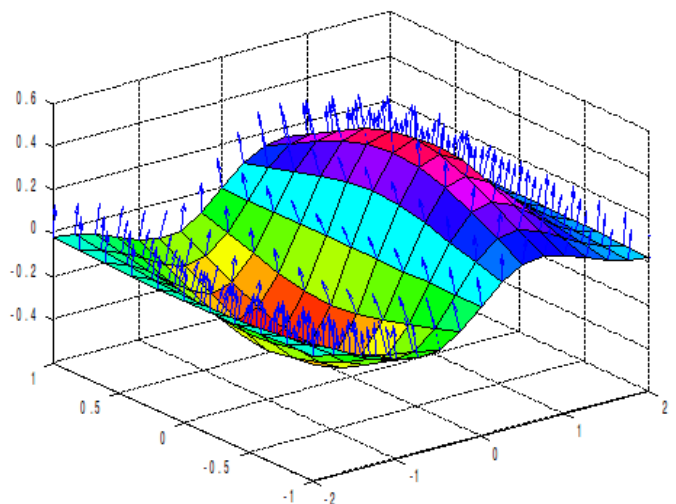
For example, to plot the field $\mathbf{F}(x, y, z) = 3x \mathbf{i} + 2y \mathbf{j} + 5z \mathbf{k}$ inside the box $-3 \leq x, y, z \leq 3$ you can use the following commands and obtain the second plot above.

[x,y,z]=meshgrid(-3:1:3, -3:1:3, -3:1:3); quiver3(x, y, z, 3*x, 2*y, 5*z) axis equal

The step size 1 in the meshgrid command was arbitrarily selected. To plot vectors more densely, decrease this step size.

One can plot the gradient field of the surface $z=f(x,y)$ by using the command **surfnorm(x,y,f(x,y))**. Note that this command requires the variables to be represented as vectors. For example, let us plot the gradient field of the surface $z=x e^{-x^2-y^2}$ together with the surface itself. Let us select the domain $-2 \leq x \leq 2$ and $-1 \leq y \leq 1$ and the step size 0.2.

[X,Y] = meshgrid(-2:0.2:2,-1:0.2:1);
Z = X.* exp(-X.^2 - Y.^2);



(represents \mathbf{Z} as a vector function of \mathbf{X} and \mathbf{Y})

[U,V,W] = surfnorm(X,Y,Z);

(calculates the gradient of $z=f(x,y)$ and stores its entries in the vector field **[U,V,W]**)

quiver3(X,Y,Z,U,V,W,0.5) (plots the gradient vector field, 0.5 at the end of the command ensures that the vectors have the same length)

hold on

surf(X,Y,Z) (plots the surface as well)

hold off

axis ([-2 2 -1 1 -.6 .6]) (for a nice plot, choose the window for x,y and z values in the end)

3.4 Special effects

You can make three-dimensional plots nicer by

- using perspective (**camproj('perspective')**; undo by **camproj('orthographic')**)
- showing bounding box (**box on**; undo by **box off**),
- making axis units equal (**axis tight, axis equal**; undo by **axis normal**),
- showing grid (**grid on**; undo by **grid off**) and
- allowing rotation by mouse dragging (**rotate3d on**; undo by **rotate3d off**).

You can produce an animated picture using the command **comet** for two-dimensional parametric curves and **comet3** for three-dimensional parametric curves. Prior to the command, you need to specify the values of your parameter indicating the domain and the step size similarly as in the **meshgrid** command. The command **comet** (or **comet3**) produces a parametric plot of a curve together with an animation that allows you to see how the curve is being traced out. For example, to trace the motion on the circle $x = \cos t$, $y = \sin t$ for $0 \leq t \leq 4\pi$, use

t = 0:0.1:4*pi; (the step size 0.1 was chosen arbitrarily)

comet(cos(t), sin(t))

If the point was moving too fast and you could have not followed it, you can reparameterize the same circle by $x = \cos t/50$, $y = \sin t/50$ for $0 \leq t \leq 200\pi$. Note that this still gives you four full rotations. Then use

t = 0:0.1:200*pi;

comet(cos(t/50), sin(t/50))

For the three-dimensional animations, use **comet3**. For example, to trace the full four spins on the helix $x = \cos t/50$, $y = \sin t/50$, $z = t$ use $0 \leq t \leq 200\pi$ and animate by

t = 0:0.1:200*pi;

comet3(cos(t/50), sin(t/50), t)

Practice problems 2

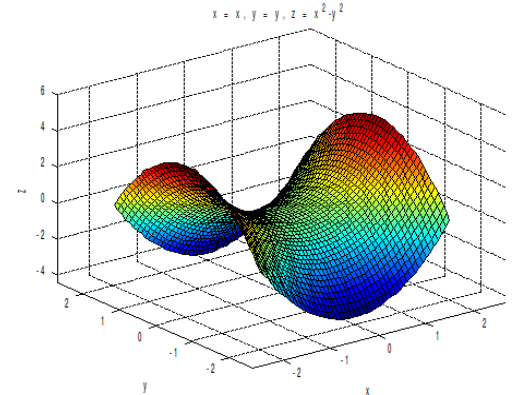
1. Graph the parametric curve $x = t \cos t$, $y = t \sin t$ for $0 \leq t \leq 10\pi$.
2. Write down the parametric representation of the curve of the intersection of the cylinder

- $y^2 + z^2 = 4$ with the plane $x + z = 3$. Graph this curve using this representation.
- Graph the hyperbolic paraboloid $z = x^2 - y^2$ over the square $-2 \leq x \leq 2, -2 \leq y \leq 2$.
 - Graph the cylinder $y^2 + z^2 = 4$ for $0 \leq x \leq 10$.
 - Graph the paraboloid $z = x^2 + y^2$ using the cylindrical coordinates and $0 \leq r \leq 3$.
 - Graph the sphere of radius 3 centered at origin.
 - Graph the vector field $\mathbf{F}(x, y, z) = y \mathbf{i} + x \mathbf{j} + z^2 \mathbf{k}$
 - If \mathbf{X} represents the vector field $x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, the gravitational vector field $\mathbf{F}(\mathbf{X})$ is given by $\mathbf{F}(\mathbf{X}) = -c \mathbf{X} / |\mathbf{X}|^3$ where c is a constant. Assuming that $c=3$, plot this vector field.
 - Trace the curve $x = t \cos(t/20), y = t \sin(t/20)$ in time for $0 \leq t \leq 200\pi$.

Solutions

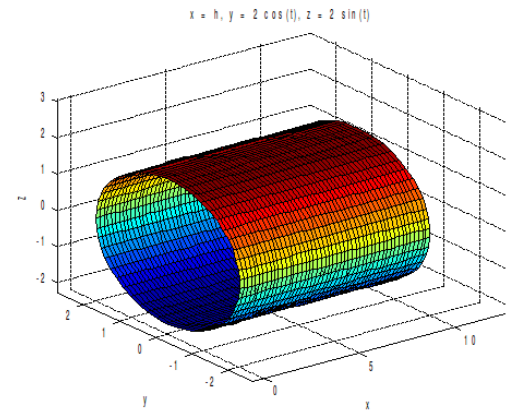
1. `ezplot('t*cos(t)', 't*sin(t)', [0, 10*pi])`

2. Since $y^2 + z^2 = 4$, you can use $y = 2 \cos t, z = 2 \sin t$. Solving the equation of the plane for x , you have $x = 3 - z$. Thus $x = 3 - 2 \sin t$. So, the curve in the intersection parametrizes as $x = 3 - 2 \sin t, y = 2 \cos t, z = 2 \sin t$. The interval $0 \leq t \leq 2\pi$ is sufficient for the whole curve to be displayed. So, graph by using `ezplot3('3-2*sin(t)', '2*cos(t)', '2*sin(t)', [0, 2*pi])`



3. Graph the hyperbolic paraboloid by `ezsurf('x', 'y', 'x^2-y^2', [-2, 2, -2, 2])`
The graph is given on the first figure.

4. The cylinder $y^2 + z^2 = 4$ can be parametrized by $x = h, y = 2 \cos t, z = 2 \sin t$. Graph it by `ezsurf('h', '2*cos(t)', '2*sin(t)', [0, 10, 0, 2*pi])`
The graph is given on the second figure.



5. Parametrize the paraboloid $z = x^2 + y^2$ using the cylindrical coordinates as $x = r \cos t, y = r \sin t, z = r^2$. Then graph it by `ezsurf('r*cos(t)', 'r*sin(t)', 'r^2', [0, 3, 0, 2*pi])`
6. Use the spherical coordinates with $r=3$ and graph by `ezsurf('3*cos(t)*sin(s)', '3*sin(t)*sin(s)', '3*cos(s)', [0, pi, 0, 2*pi])` followed by `axis equal`.
7. Since the window was not specified, use any values in the `meshgrid` command. For example `[x,y,z]=meshgrid(-3:1:3, -3:1:3, -3:1:3)`; then use `quiver3(x, y, z, y, x, z^2)` and `axis equal` for a nicer graph.
8. If \mathbf{X} is the field $x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, then $|\mathbf{X}|$ is given by $(x^2 + y^2 + z^2)^{1/2}$. So the coordinates of the gravitational field \mathbf{F} are given by $-3x/(x^2 + y^2 + z^2)^{3/2}, -3y/(x^2 + y^2 + z^2)^{3/2}$ and $-3z/(x^2 + y^2 + z^2)^{3/2}$. You can plot this field by the following. The values in the `meshgrid` command are arbitrarily selected.

```
[x,y,z]=meshgrid(-4:2:4, -4:2:4, -4:2:4);
U = inline(vectorize('-x/(x^2+y^2+z^2)^(3/2)'), 'x', 'y', 'z');
V = inline(vectorize('-y/(x^2+y^2+z^2)^(3/2)'), 'x', 'y', 'z');
W = inline(vectorize('-z/(x^2+y^2+z^2)^(3/2)'), 'x', 'y', 'z');
P = U(x,y,z); Q = V(x,y,z); R = W(x,y,z);
quiver3(x, y, z, P, Q, R)
axis equal
```

9. `t = 0:0.1:200*pi; comet(t*cos(t/20), t*sin(t/20))`

4. Integration of Multi-variable Functions

The command that evaluates a double integral of a function $f(x,y)$ for $a \leq x \leq b$ and $c(x) \leq y \leq d(x)$ has the following form.

`int(int(f(x,y), y, c(x), d(x)), x, a, b)`

Similarly, the command that evaluates a triple integral of a function $f(x,y,z)$ for $a \leq x \leq b$, $c(x) \leq y \leq d(x)$, and $e(x,y) \leq z \leq f(x,y)$ has the following form.

`int(int(int(f(x,y,z), z, e(x,y), f(x,y)), y, c(x), d(x)), x, a, b)`

For example, we can evaluate the integral $\int_{-1}^1 \int_{2x^2}^{1+x^2} (x+2y) dy dx$ as follows.

`syms x y`

`int(int(x+2*y, y, 2*x^2, 1+x^2), x, -1, 1) ans = 32/15`

Similarly, we can evaluate the triple integral $\int_0^1 \int_0^x \int_0^{xy} 6xyz dx dy dz$ as follows.

`int(int(int(6*x*y*z, z, 0, x*y), y, 0, x), x, 0, 1) ans = 3/32`

In some cases, you have to be careful of the order of integration. For example, to evaluate the

integral $\int_0^1 \int_y^{y+z} \int_y^{2y} 2y \sin x dx dy dz$ note that you need to integrate with the respect to x first (because of the bound with both y and z), then with respect to z (because of the middle integral with y in the bounds) and, lastly, with respect to y . So, the integral can be evaluated by

`int(int(int(2*y*sin(x), x, y, y+z), z, y, 2*y), y, 0, 1)`

Practice problems 3

1. Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the region bounded by $y = x^2$ and $x = y^2$.

2. Evaluate the integral $\int_0^1 \int_0^z \int_0^{x+z} 6xzy dz dx$.

Solutions

- Note first that the curves $y = x^2$ and $x = y^2$ intersect when $x = x^4$. Thus $x(1-x^3)=0$ which produces two solutions $x=0$ and $x=1$ (alternative use `solve('x-x^4=0','x')`). When $0 \leq x \leq 1$, \sqrt{x} is the upper and x^2 the lower bound for y . So the volume can be calculated by integrating $x^2 + y^2$ for $0 \leq x \leq 1$ and $x^2 \leq y \leq \sqrt{x}$. `int(int(x^2+y^2, y, x^2, sqrt(x)), x, 0, 1)`
- Integrate with respect to y first, then with respect to x and, finally, with respect to z . `int(int(int(6*x*z, y, 0, x+z), x, 0, z), z, 0, 1)`

5. Series and Taylor Series

You can evaluate both finite and infinite sums in Matlab using the command `symsum` which has the following form.

`symsum(n-th term of the sum, initial n-value, final n-value)`

For example, to find the numerical value of $\sum_{n=1}^{10} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ we can use the following.

```
syms n
symsum(1/n^2, 1, 10)      ans = 1968329/1270080
vpa(ans, 3)              ans = 1.54
symsum(1/n^2, 1, inf)    ans = 1/6*pi^2
vpa(ans, 3)              ans = 1.64
```

You can work with symbolic series as well. For example, to evaluate $\sum_{n=0}^{\infty} a^n$ where $-1 < a < 1$

you can use `syms n a`
`symsum(a^n, 0, inf) ans = -1/(a-1)`

The command `taylor` generates the Taylor polynomial of specified order n centered at 0 by `taylor(function f(x), order n)`

and centered at a specified point $x=a$ by `taylor(function f(x), order n, point a)`

For example, for the Taylor polynomial of e^x of order 4 centered at 0, you can use

```
syms x
taylor(exp(x), 4)        ans = 1+x+1/2*x^2+1/6*x^3
```

For the Taylor polynomial of e^x of order 4 centered at 3, you can use

```
taylor(exp(x), 4, 3)    ans= exp(3)+exp(3)*(x-3)+1/2*exp(3)*(x-3)^2+1/6*exp(3)*(x-3)^3
```

Practice problems 4

- Determine whether the following series is convergent or divergent using Matlab.

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n} \quad \sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$$

2. Find the Taylor polynomial of order 5 at 0 and at 2 for the function $\sin x$.

Solutions

1. For the first series use **`symsum(n^2/2^n, 1, inf)`** since the answer is a finite number, the series is convergent. For the second series use **`symsum((n-1)/(n^2+n), 1, inf)`** and obtain the answer **`Inf`** indicating that the series is divergent.
2. Centered at 0: **`taylor(sin(x), 5)`** or **`taylor(sin(x), 5, 0)`**.
Centered at 2: **`taylor(sin(x), 5, 2)`**.