Calculus 3
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Review for Exam 2

1. **Maximum and Minimum Values.** Find the maximum and minimum values of f.

   (a) $z = 9 - 2x + 4y - x^2 - 4y^2$ (b) $z = x^2 + y^2 + x^2y + 4$
   (c) $z = \frac{x^2y^2-8x+y}{xy}$ (d) $z = xy - 2x - y$

2. **Lagrange Multipliers.** Find the maximum and minimum values of f subject to the given constraint(s).

   (a) $f(x, y) = x^2 - y^2;\quad x^2 + y^2 = 1$ (b) $f(x, y) = x^2y;\quad x^2 + 2y^2 = 6$
   (c) $f(x, y, z) = 2x + 6y + 10z;\quad x^2 + y^2 + z^2 = 35$
   (d) $f(x, y, z) = 3x - y - 3z;\quad x + y - z = 0;\quad x^2 + 2z^2 = 1$

3. **Applications.**

   (a) Find the points on the surface $z^2 = xy + 1$ that are closest to the origin.
   (b) Set up a system of equations for finding the dimensions of the rectangular box with the largest volume if the total surface area is 64 cm^2.
   (c) A cardboard box without a lid is to have volume of 32,000 cm^3. Set up a system of equations for finding the dimensions that minimize the amount of cardboard used.

4. **Double Integrals.** Find the following.

   (a) $\int\int_D (x + 2y)dxdy$ where $D = \{\ (x, y) \mid 0 \leq x \leq 1,\ \ 0 \leq y \leq x^2\ \}$.
   (b) $\int\int_D 2xdxdy$ where $D = \{\ (x, y) \mid 0 \leq y \leq 1,\ \ y \leq x \leq e^y\ \}$.
   (c) $\int\int_D y^3dxdy$ where D is the triangular region with vertices (0, 2), (1, 1) and (3, 2).
   (d) $\int\int_D xdxdy$ where D is the *right half* of the disk centered at the origin of radius 5.
   (e) $\int\int_D xydxdy$ where D is the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 25$ in *the first quadrant.*
   (f) $\int\int_D \frac{1}{\sqrt{x^2+y^2}}dxdy$ where D is the region inside $r = 4\cos\theta$ and outside $r = 2$.
   (g) Find the average value of the function $f(x, y) = 4x$ on the region D between the parabolas $y = x^2 - 2$ and $y = 3x - x^2$.
   (h) Find the mass and the center of mass of the lamina that occupies the triangular region with vertices (0,0), (2,1) and (0,3) and has the density function $\rho(x, y) = x + y$.

5. **The Volume.**

   (a) Find the volume of the solid bounded by the plane $2x + 2y + z = 4$ in the first octant.
   (b) Find the volume of the solid under the surface $z = xy$ and above the triangle with vertices (1, 1), (4, 1) and (1, 2).

(c) Find the volume of the solid under the paraboloid $z = x^2+y^2$ and above the disk $x^2+y^2 \leq 9$.

(d) Find the volume of the solid above the cone $z = \sqrt{x^2+y^2}$ and below the paraboloid $z = 2 - x^2 - y^2$.

(e) Find the volume of the solid enclosed by the paraboloids $z = x^2+y^2$ and $z = 36-3x^2-3y^2$.

(f) Find the volume of the solid bounded by the cylinder $x^2+y^2 = 1$ and the planes $z = 2-y$ and $z = 0$ in the first octant.

(g) Find the volume of the solid bounded by the cylinder $x^2+y^2 = 1$ and the planes $z = 2-y$ and $z = 0$ and to the right of the xz-plane.

6. **Surface Area.**

(a) Find the area of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.

(b) Find the area of the part of the surface $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(c) Find the area of the part of the paraboloid $z = x^2 + y^2$ below the plane $z = 4$ and in the first octant.

Solutions

More detailed solutions of the problems can be found on the class handouts.

1. Maximum and Minimum Values. (a) Maximum $f(-1, 1/2) = 11$ (b) Minimum $f(0,0) = 4$, saddle points $(\pm\sqrt{2}, -1)$ (c) Maximum $f(-1/2, 4) = -6$ (d) Saddle point at $(1, 2)$

2. Lagrange Multipliers. (a) Max. $f(\pm 1, 0) = 1$, min. $f(0, \pm 1) = -1$ (b) Max. $f(\pm 2, 1) = 4$, min. $f(\pm 2, -1) = -4$ (c) Max. $f(1, 3, 5) = 70$, min. $f(-1, -3, -5) = -70$ (d) Max. $f(2/\sqrt{6}, -3/\sqrt{6}, -1/\sqrt{6}) = 12/\sqrt{6} = 2\sqrt{6}$, min. $f(-2/\sqrt{6}, 3/\sqrt{6}, 1/\sqrt{6}) = -12/\sqrt{6} = -2\sqrt{6}$.

3. Applications. (a) $(0, 0, \pm 1)$

   (b) Equations: $yz - 2\lambda y - 2\lambda z = 0$, $xz - 2\lambda x - 2\lambda z = 0$, $xy - 2\lambda x - 2\lambda y = 0$, $2xy + 2yz + 2xz = 64$. If solved, the equations would yield: $x = y = z = 4\sqrt{6}/3$ cm.

   (c) Equations: $y + 2z - yz\lambda = 0$, $x + 2z - xz\lambda = 0$, $2x + 2y - xy\lambda = 0$, $xyz = 32,000$. (If solved, the equations would yield: square base of side $x = y =$40 cm, height $z =$20 cm.)

4. Double Integrals. (a) $\frac{9}{20}$ (b) 2.86 (c) $\frac{147}{20}$ (d) $\frac{250}{3}$ (e) $\frac{609}{8}$ (f) $4\sqrt{3} - 4\pi/3$ (g) 3 (h) mass = 6, center of mass = $(\frac{3}{4}, \frac{3}{2})$

5. The Volume. (a) $\frac{8}{3}$ (b) $\frac{31}{8}$ (c) $\frac{81\pi}{2}$ (d) $\frac{5\pi}{6}$ (e) 162π (f) 1.24. (g) 2.47.

6. Surface Area. (a) $3\sqrt{14}$ (b) 30.85 (c) 9.04