Fundamentals of Mathematics
Lia Vas

Formulas for Exam 1

1. **The truth tables for logical connectives.**

p	$\neg p$
$\top$	$\bot$
$\bot$	$\top$

p	q	$p \wedge q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$

p	q	$p \vee q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$

p	q	$p \Rightarrow q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\top$

p	q	$p \Leftrightarrow q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\top$

2. **Forms of an implication.** "If p, then q" is $p \Rightarrow q$.

   "q only if p" is $q \Rightarrow p$. "p is necessary for q" is $q \Rightarrow p$. "p is sufficient for q" is $p \Rightarrow q$.

3. **Tautologies of propositional logic.**

$p \vee \neg p$	the law of excluded middle
$p \wedge (p \Rightarrow q) \Rightarrow q$	Modus Ponens
$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	associativity for $\wedge$
$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$	associativity for $\vee$
$p \wedge q \Leftrightarrow q \wedge p$	commutativity for $\wedge$
$p \vee q \Leftrightarrow q \vee p$	commutativity for $\vee$
$\neg\neg p \Leftrightarrow p$	double negation
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	De Morgan's laws
$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	
$p \wedge p \Leftrightarrow p$	Idempotent law for $\wedge$
$p \vee p \Leftrightarrow p$	Idempotent law for $\vee$
$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributivity for $\wedge$ and $\vee$
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	Distributivity for $\vee$ and $\wedge$
$(p \Leftrightarrow q) \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)$	biconditional law
$(p \Rightarrow q) \Leftrightarrow \neg p \vee q$	Material Implication
$\neg(p \Rightarrow q) \Leftrightarrow p \wedge \neg q$	the negation of an implication
$(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$	contrapositive
$(p \Rightarrow (q \Rightarrow r)) \Leftrightarrow (p \wedge q \Rightarrow r)$	exportation

4. **Tautologies of predicate logic.**

$(\forall x)P(x) \Leftrightarrow P(x)$	universal quantification of a variable
$(\forall x)(\forall y)P(x,y) \Leftrightarrow (\forall y)(\forall x)P(x,y)$	the order of the quantifiers
$(\exists x)(\exists y)P(x,y) \Leftrightarrow (\exists y)(\exists x)P(x,y)$	
$(\exists x)(\forall y)P(x,y) \Rightarrow (\forall y)(\exists x)P(x,y)$	
$(\forall x)P(x) \Leftrightarrow (\forall y)P(y)$	renaming the variables
$(\exists x)P(x) \Leftrightarrow (\exists y)P(y)$	
$\neg(\forall x)P \Leftrightarrow (\exists x)\neg P$	moving $\neg$ through quantifiers
$\neg(\exists x)P \Leftrightarrow (\forall x)\neg P$	
$(\forall x)(P \wedge Q) \Leftrightarrow ((\forall x)P) \wedge ((\forall x)Q)$	moving quantifiers through $\wedge$ and $\vee$
$(\exists x)(P \vee Q) \Leftrightarrow ((\exists x)P) \vee ((\exists x)Q)$	
$(\forall x)(P \vee Q) \Leftarrow ((\forall x)P) \vee ((\forall x)Q)$	
$(\exists x)(P \wedge Q) \Rightarrow ((\exists x)P) \wedge ((\exists x)Q)$	

5. **Restricted quantification.**

$$(\forall x : A(x))P(x) \text{ shortens } (\forall x)(A(x) \Rightarrow P(x))$$

$$(\exists x : A(x))P(x) \text{ shortens } (\exists x)(A(x) \wedge P(x))$$