Fundamentals of Mathematics
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Formulas for Exam 3

1. **Cardinality.**

$$A \approx B \quad \Leftrightarrow \quad (\exists f : A \to B) f \text{ is a bijection.}$$

In this case, $|A| = |B|$.

Cardinal adition:

$$|A| + |B| = |(A \times \{\square\}) \cup (B \times \{\triangle\})|$$

If $A \cap B = \emptyset$, then

$$|A| + |B| = |A \cup B|$$

Cardinal multiplication:

$$|A| \cdot |B| = |A \times B|.$$

Some cardinal identities. $n \in \omega$ is a finite cardinal and α is an infinite cardinal.

$$\alpha + n = \alpha \qquad \alpha + \alpha = \alpha$$

$$\alpha \cdot n = \alpha \qquad \alpha \cdot \alpha = \alpha$$

2. **Mathematical Induction.**

"Basic" Induction for showing that $P(n)$ holds for any $n \in \omega$.
1. $P(0)$ holds.
2. If $P(n)$ holds, then $P(n+1)$ holds.

Limited Induction for showing that $P(n)$ holds for any $n \in \omega, n \geq k$.
1. $P(k)$ holds.
2. If $P(n)$ holds for $n \geq k$, then $P(n+1)$ holds.

Complete induction for showing that $P(n)$ holds for any $n \in \omega$
1. $P(0)$ holds.
2. If $P(k)$ holds for all $k \leq n$, then $P(n+1)$ holds.

Double Induction for showing that $P(m, n)$ holds for all $m, n \in \omega$.

1. Show that $P(0, n)$ holds by showing
 1a. $P(0, 0)$ holds.
 1b. If $P(0, n)$, then $P(0, n+1)$ holds.
2. Assuming that $P(m, n)$ holds, show that $P(m+1, n)$ holds.

Alternatively,

1. Show that $P(m, 0)$ holds by showing
 1a. $P(0, 0)$ holds.
 1b. If $P(m, 0)$, then $P(m+1, 0)$ holds.
2. Assuming that $P(m, n)$ holds, show that $P(m, n+1)$ holds.

For $n \in \omega$, **factorial** function is given by $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$, or, recursively, by

$$0! = 1, \qquad (n+1)! = n! \cdot (n+1).$$