General Calculus
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Formulas for the Final Exam

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$
$\int y\, dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln\|x\|$	$-\cos x$	$\sin x$

3. **Rules of Differentiation.**

   (a) Product rule.

$$\text{If} \quad y = f \cdot g, \quad \text{then} \quad y' = f' \cdot g + g' \cdot f$$

   (b) Quotient rule.

$$\text{If} \quad y = \frac{f}{g}, \quad \text{then} \quad y' = \frac{f' \cdot g - g' \cdot f}{g^2}$$

   (c) Chain rule.

$$\text{If} \quad y = f(g(x)), \quad \text{then} \quad y' = f'(g(x)) \cdot g'(x)$$

4. **Definition of Derivative.** Derivative of $y = f(x)$ at $x = a$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

In Leibniz notation, $\frac{dy}{dx}\Big|_{x=a} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$.

5. **Average and instantaneous rate of change.**

   (a) The average rate of change of $f(x)$ over $[a, b]$:

$$\frac{f(b) - f(a)}{b - a}$$

   (b) The instantaneous rate of change of $f(x)$ at $x = a$: $\quad f'(a)$.

6. **Linear Approximation.**

$$f(a + dx) \approx f(a) + f'(a)dx \quad \text{or} \quad f(x) \approx f(a) + f'(a)(x - a)$$

7. **Tangent Line.** $y_0 = f(x_0)$, $m = f'(x_0)$

$$y - y_0 = m(x - x_0)$$

8. **Velocity and Distance traveled.** The velocity $v(t) = s'(t) = \frac{ds}{dt}$ and $s(t) = \int v(t)\, dt$. The acceleration $a(t) = v'(t) = \frac{dv}{dt} = s''(t) = \frac{d^2s}{dt^2}$.

9. **Left and Right Sum.**

$$\text{Left sum} = \frac{b-a}{n}\ \left(f(x_0) + f(x_1) + \ldots + f(x_{n-1})\right)$$

$$\text{Right sum} = \frac{b-a}{n}\ \left(f(x_1) + f(x_2) + \ldots + f(x_n)\right)$$

10. **Area between $f(x)$ and x-axis for $a < x < b$.**

    - If $f(x) > 0$ for $a < x < b$, then area is $\int_a^b f(x)\, dx$
    - If $f(x) < 0$ for $a < x < b$, then area is $-\int_a^b f(x)\, dx$
    - If $f(x) < 0$ for $a < x < c$ and $f(x) > 0$ for $c < x < b$, then area is $-\int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

    Area between $f(x)$ and $g(x)$. if $f(x) > g(x)$ for $a < x < b : \int_a^b (f(x) - g(x))\, dx$