General Calculus
Lia Vas

Review for Exam 1

1. **Limits.** Evaluate the following limits.

(a) $\lim_{x\to 1} \dfrac{x-1}{x^2-3x+2}$ (b) $\lim_{x\to \infty} \dfrac{x-1}{x^2-3x+2}$

(c) $\lim_{x\to 3} \dfrac{x^2-x-6}{x^2-2x-3}$ (d) $\lim_{x\to \infty} \dfrac{x^2-x-6}{x^2-2x-3}$

(e) $\lim_{x\to 2} \sqrt{3x^2-5x+2}$ (f) $\lim_{x\to 0} 5^x+3$

(g) $\lim_{x\to -\infty} 5^x+3$ (h) $\lim_{x\to \infty} 5^x+3$

(i) $\lim_{x\to 3^-} \dfrac{2}{x-3}$ (j) $\lim_{x\to 3} \dfrac{2}{x-3}$

(k) $\lim_{x\to \infty} \dfrac{2}{x-3}$ (l) $\lim_{x\to \infty} \dfrac{2x}{x-3}$

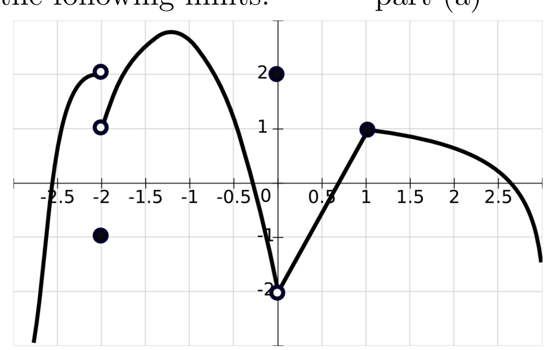
(m) $\lim_{x\to 0^+} \dfrac{x+2}{x(x-2)}$ (n) $\lim_{x\to \infty} \dfrac{x+2}{x(x-2)}$

(o) $\lim_{x\to -\infty} 3^{\frac{4}{x-2}}-5$ (p) $\lim_{x\to 2^-} 3^{\frac{4}{x-2}}-5$ (q) $\lim_{x\to 2^+} 3^{\frac{4}{x-2}}-5$

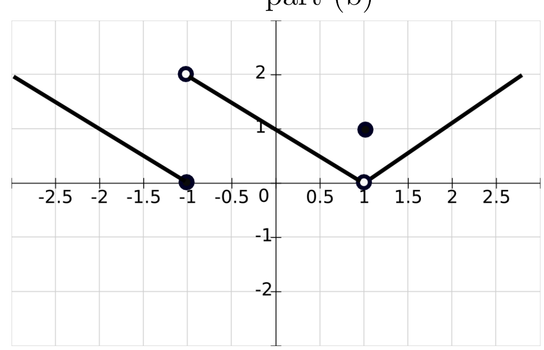
(r) $\lim_{x\to -1^+} \ln(x+1)+3$ (s) $\lim_{x\to \infty} \ln(x+1)+3$ (t) $\lim_{x\to \infty} \cos\frac{x-1}{x^2}$

2. **Limits from graphs.** Consider the function $f(x)$ given by the following graph. Determine the following limits.

part (a)



part (b)



(a) $\lim_{x\to -2^-} f(x)$ $\lim_{x\to -2^+} f(x)$ $\lim_{x\to -2} f(x)$ $f(-2)$ $\lim_{x\to 0} f(x)$ $f(0)$ $\lim_{x\to 1^-} f(x)$ $\lim_{x\to 1^+} f(x)$

(b) $\lim_{x\to -1^-} f(x)$ $\lim_{x\to -1^+} f(x)$ $\lim_{x\to -1} f(x)$ $f(-1)$ $\lim_{x\to 1^-} f(x)$ $\lim_{x\to 1^+} f(x)$ $\lim_{x\to 1} f(x)$ $f(1)$

3. **Asymptotes.** Find the horizontal and vertical asymptotes (if any) of the following functions.

(a) $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - x - 6}$ (b) $f(x) = \dfrac{x+5}{x^2 - x - 6}$ (c) $f(x) = \dfrac{x^3+1}{x(3-x)}$

4. **Continuity.** Determine if the following functions are continuous at given points.

(a) $f(x) = \begin{cases} x+2 & x < -1 \\ x+1 & -1 \le x < 1 \\ 3-x & x \ge 1 \end{cases}$ $x = -1$ and $x = 1$. (b) $f(x) = \begin{cases} x^2 & x < 0 \\ x & 0 < x < 2 \\ 1 & x = 2 \\ 4-x & x > 2 \end{cases}$

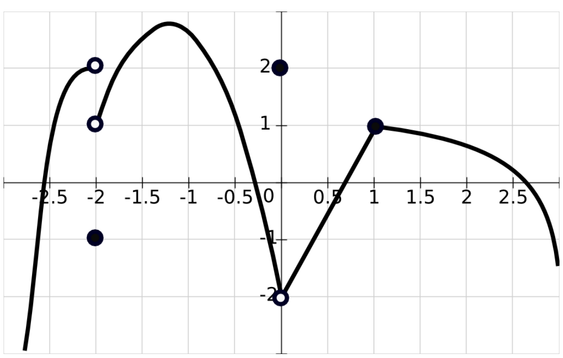
$x = 0$ and $x = 2$.

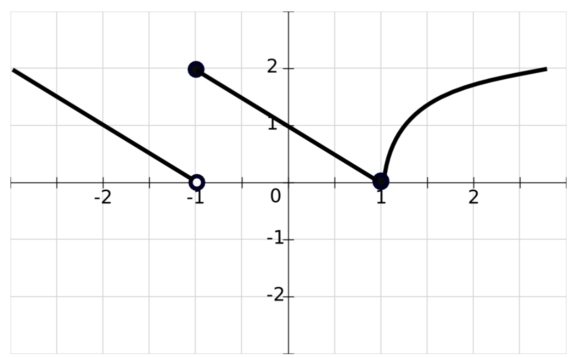
5. **Continuity and Differentiability.** Discuss the continuity and differentiability of the following functions at every point.

(a) $f(x) = x^2 + 2$ (b) $f(x) = 5\sqrt[3]{x^2}$ (c) $f(x) = 2 - 3x^{1/5}$

(d) The function given by the graph on the left.

(e) The function given by the graph on the right.





6. **Derivative definition.** Find the derivative of the following functions at a given point using the definition of derivative at a point.

(a) $f(x) = x^2 - 3x,\ x = 2$ (b) $f(x) = x^2 - 3x$, any x.

7. **Finding Derivative.** Find the first derivative of the given functions. For part (a) find the first five derivatives.

(a) $y = 2x^5 - 3x^3 + 5x - 9$ (b) $y = \frac{x^3}{2} + \sqrt{x^3}$ (c) $y = \frac{4}{x^2} - \frac{1}{3x^6}$

8. **Average and instantaneous rate of change.**

(a) Let $f(x) = \sqrt{x^5} - 10\sqrt{x}$. Find the average rate of change over $[0, 4]$. Find the instantaneous rate of change at $x = 4$.

(b) Let $f(x) = x + \frac{4}{x}$. Find the average rate of change over $[1, 2]$. Find the instantaneous rate of change at $x = 1$.

9. **Tangent Line.** Find an equation of the line tangent to the graph of the given equation at the indicated point.

(a) $f(x) = x + 3 - \frac{5}{x}$ at $x = 1$.

(b) $f(x) = \frac{2}{x} + \frac{x}{2}$ at $x = 2$.

(c) $f(x) = \sqrt{x^3} + \sqrt[3]{x^2}$ at $x = 1$.

10. **Applications.**

(a) The function $B(t) = \frac{2 \cdot 10^7}{1+7e^{-3t/10}}$ models the biomass (total mass of the members of the population) in kilograms of a Pacific halibut fishery after t years. Determine the biomass in the long run.

(b) Brine that contains the solution of water and salt is pumped into a water tank. The concentration of salt is increasing according to the formula $C(t) = \frac{5t}{100+t}$ grams per liter. Determine the concentration of salt after a substantial amount of time.

(c) If we approximate the gravitational acceleration g by 9.8 meters per seconds squared, the distance from the initial height of an object dropped from it to the ground can be described as $s(t) = \frac{g}{2}t^2 \approx 4.9t^2$. (a) Find the average velocity of the object in the first three seconds. (b) Find the velocity of the object three seconds into the fall. (c) Find the formula computing the velocity at any point t.

(d) A company determines that its cost function is $C(x) = 1000 + 35x - .01x^2$, $0 \leq x \leq 300$, where x is the number of items produced and $C(x)$ is the cost of producing x items in dollars. Find the average rate of change in cost when x is changing from 100 to 150. Then, find the instantaneous rate of change in cost when producing 200 units and estimate the cost of producing 201 items.

(e) Assume that the mathematical model for the growth of a locust tree in its first century of life is given by $h(t) = 3\sqrt{t}$, $0 \leq t \leq 100$, where t is the age of the tree in years and $h(t)$ is the height of the tree in feet. Find $h(64)$ and $h'(64)$ and interpret the meaning of your answers in a full sentence.

(f) The mass of a bacteria culture t hours after the start of experiment, is modeled by $N(t) = 3t^{5/2}$, in milligrams. (a) Determine the mass 16 hours after experiment started. (b) Determine how fast the mass of bacteria increases 9 hours after the experiment started. (c) Determine the time when the mass is 300 mg.

(g) The body mass index (BMI) is a number obtained as $BMI = \frac{703w}{h^2}$ where w is the weight in pounds and h is the height in inches. For a 125-lb female that is now 65 inches tall but growing, calculate how fast is BMI changing with each new inch. Explain the meaning of the answer.

Solutions

Solutions below contain only the final answer. See the class handouts for more step by step solutions.

1. Limits. (a) -1 (b) 0 (c) $\frac{5}{4}$ (d) 1 (e) 2 (f) 4 (g) 3 (h) ∞ (i) $-\infty$ (j) does not exist (k) 0 (l) 2 (m) $-\infty$ (n) 0 (o) -2 (p) -5 (q) ∞ (r) $-\infty$ (s) ∞ (t) $\cos 0 = 1$

2. Limits from graphs. (a) $\lim_{x\to -2^-} f(x) = 2, \lim_{x\to -2^+} f(x) = 1$, $f(-2) = -1$, $\lim_{x\to -2} f(x)$ does not exist, $\lim_{x\to 0} f(x) = -2$, $f(0) = 2$, $\lim_{x\to 1^-} f(x) = 1$, and $\lim_{x\to 1^+} f(x) = 1$.

(b) $\lim_{x\to -1^-} f(x) = 0, \lim_{x\to -1^+} f(x) = 2, f(-1) = 0, \lim_{x\to -1} f(x)$ does not exist, $\lim_{x\to 1^-} f(x) = 0, \lim_{x\to 1^+} f(x) = 0,\ \lim_{x\to 1} f(x) = 0,\ f(1) = 1.$

3. Asymptotes. (a) $x = 3$ is the only vertical asymptote and $y = 1$ is a horizontal asymptote. (b) $x = 3$ and $x = -2$ are vertical asymptotes and $y = 0$ is a horizontal asymptote. (c) $x = 3$ and $x = 0$ are vertical asymptotes and $f(x)$ does not have a horizontal asymptote.

4. Continuity. (a) Not continuous at -1. Continuous at 1. (b) Not continuous at 0 and at 2.

5. Continuity and Differentiability. (a) $f'(x) = 2x$, defined at every x-value so $f(x)$ is differentiable (thus continuous too) for every x.

   (b) $f'(x) = \frac{10}{3\sqrt[3]{x}}$ defined for every value of x except $x = 0$. There has a corner at $x = 0$ so $f(x)$ is differentiable for every $x \neq 0$. The function is continuous at every point.

   (c) $f'(x) = \frac{-3}{5\sqrt[5]{x^4}}$ defined for every value of x except $x = 0$. There is a vertical tangent at $x = 0$ so $f(x)$ is differentiable for every $x \neq 0$. The function is continuous at every point.

   (d) The function is differentiable at every point different from $-2, 0$ and 1. At $x = -2$ and $x = 0$ the function is not differentiable since it is not continuous (a jump at -2 and a hole at 0). At $x = 1$ the function is not differentiable since there is a corner in the graph but it is continuous.

   (e) The function is differentiable at every point different from -1 and 1. At $x = -1$ the function is not differentiable since it is not continuous because of a break. At $x = 1$ the function is not differentiable since there is a corner in the graph but it is continuous.

6. Derivative Definition. (a) 1 (b) $f'(x) = 2x - 3$

7. Finding Derivative. (a) $f(x) = 2x^5 - 3x^3 + 5x - 9 \Rightarrow f'(x) = 10x^4 - 9x^2 + 5 \Rightarrow f''(x) = 40x^3 - 18x \Rightarrow f'''(x) = 120x^2 - 18 \Rightarrow f^{(4)}(x) = 240x \Rightarrow f^{(5)}(x) = 240.$

   (b) $y' = \frac{3}{2}x^2 + \frac{3}{2}\sqrt{x}$ (c) $y' = -\frac{8}{x^3} + \frac{2}{x^7}$

8. Average and instantaneous rate of change. (a) 3 and 17.5 (b) -1 and -3.

9. Tangent Line. (a) $y = 6x - 7$ (b) $y = 2$ (c) $y = \frac{13}{6}x - \frac{1}{6}$

10. Applications.

    (a) $2 \cdot 10^7$ kilograms. (b) 5 grams per liter.

    (c) (a) 14.7 meters per second. (b) 29.4 meters per second. (c) $9.8t$ meters per second.

    (d) When production changes from 100 to 150 items produced, the cost increased at an average rate of \$32.5 per item produced. When producing 200 items, the cost is increasing at a rate of about \$31 per item produced. $C(201) \approx 7631$.

    (e) $h(64) = 24$. 64 years after it starts growing, the tree is 24 feet tall. $h'(64) = 3/16 = .1875 \approx 0.19$. 64 years after it starts growing, the tree is growing at the rate of .19 feet per year.

    (f) (a) 3072 mg. (b) 202.5 mg per hour. (c) 6.31 hours

    (g) The value of the derivative of $\frac{703(125)}{h^2}$ at $h = 65$ is $-.6399 \approx -.64$. Thus, the BMI is decreasing by .64 per inch.