Differential Equations
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Formulas for Exam 3

1. Definition of the Laplace Transform of $f(t)$.

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)dt$$

2. Table of Laplace transforms.

Function $f(t)$	**Laplace transform** $F(s)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
y'	$s\mathcal{L}[y] - y(0)$
y''	$s^2\mathcal{L}[y] - sy(0) - y'(0)$
$y^{(n)}$	$s^n\mathcal{L}[y] - s^{n-1}y(0) - \ldots - sy^{(n-2)}(0) - y^{(n-1)}(0)$
$u_c(t)$	$e^{-cs}\frac{1}{s}$
$u_c(t)f(t-c)$	$e^{-cs}\,F(s)$
$\delta(t-c)$	e^{-cs}
$f(t) * g(t)$	$F(s)\,G(s)$

3. Definitions of step and boxcar functions.

The **unit step function**

$$u_c(t) = \begin{cases} 1, & t \geq c, \\ 0, & t < c. \end{cases}$$

The **boxcar function** of height A defined by

$$\begin{cases} A, & a \leq t < b, \\ 0, & t < a \text{ and } t \geq b. \end{cases}$$

can be represented as $Au_a(t) - Au_b(t)$.

4. The **convolution** $f(t) * g(t)$ is defined as

$$f(t) * g(t) = \int_0^t f(t-\tau)g(\tau)d\tau = \int_0^t f(\tau)g(t-\tau)d\tau$$