

Review of Matlab for Differential Equations

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1. Basic Arithmetic

You can use +, -, *, \ and ^ to add, subtract, multiply, divide or exponentiate, respectively. For example if you enter:

```
>> 2^3 - 2*2
```

Matlab gives you the answer:

```
ans = 4
```

If you want to perform further calculations with the answer, you can type **ans** rather than retyping the whole answer. For example,

```
>> sqrt(ans)
```

```
ans = 2
```

To perform symbolic calculations in Matlab, use **syms** to declare the variables you plan to use. For example, suppose that you need factor x^2-3x+2 . First you need

```
>> syms x (you are declaring that x is a variable)
```

Then you can use the command **factor**.

```
>> factor(x^2-3*x+2)
```

```
ans = (x-1)*(x-2)
```

Note that we entered **3*x** to represent $3x$ in the command above. **Entering * for multiplication is always necessary in Matlab.**

Practice problems 1

1. Factor $x^3+3x^2y+3xy^2+y^3$. 2. Evaluate the following. a) $\sin(\pi/6)$ b) $\frac{\sqrt{5}+3}{\sqrt{3}-1}$ c) $\log_2(5)$

Solutions

1. **syms x y** followed by **factor(x^3+3*x^2*y+3*x*y^2+y^3)** gives you **ans=(x+y)^3**
2. Enter **sin(pi/6)**. Then **ans=.5** b) Enter **(sqrt(5)+3)/(sqrt(3)-1)**. Then **ans=7.152** c) Enter **log(5)/log(2)**. Then **ans=2.3219**.

2. Solving Equations

For solving equations, you can use the command **solve**. The command solve is always followed by parenthesis. After that, you type the equation you would like to solve in single quotes. Then type coma, and variable for which you are solving in (single) quotes. Thus, the command solve has the following form

solve('equation', 'variable for which you are solving')

For example, to solve the equation x^3-2x-4 , you type:

```
>> solve('x^3-2*x-4=0')
```

and get the following answer:

```
ans = [ 2] [-1+i] [-1-i]
```

Here i stands for the imaginary number $\sqrt{-1}$. This answer tells us that there is just one real solution, 2.

Matlab can give you both symbolic and numerical answer. For example, let us solve the equation $3x^2-8x+2=0$.

```
>> solve('3*x^2-8*x+2=0','x')
```

```
ans = [ 4/3+1/3*10^(1/2)] [ 4/3-1/3*10^(1/2)]
```

If we want to get the answer in the decimal form with, say, three significant digits, we can use the command **vpa**.

```
>> vpa(ans, 3)
```

```
ans = [ 2.38] [ 0.28]
```

The command **vpa** has the general form

vpa(expression you want to approximate, number of significant digits)

You can solve more than one equation simultaneously. For example suppose that we need to solve the system $x^2 + x + y^2 = 2$ and $2x - y = 2$. We can use:

```
>> [x,y]=solve('x^2+ x+ y^2 = 2', '2*x-y = 2')
```

to get the answer

```
x = [ 2/5] [ 1]
y = [-6/5] [ 0]
```

Note that the **[x,y]=** part at the beginning of the command was necessary since without it Matlab produces the answer:

```
ans = x: [2x1 sym] y: [2x1 sym]
```

This answer tells us just that the solutions are two values of the pair (x,y) but we do not get the solutions themselves. To get the solution vectors displayed, we must use **[x,y]=** before the command **solve**.

You can solve an equation in two variables for one of them. For example the command

```
>> solve('y^2-5*x*y+6*x^2+x=2', 'y')
```

solves the given equation for values of y in terms of x. The answer is:

ans = [3*x+2] [2*x-1]

Practice problems 2

- Solve the following equations and express the answers as decimal numbers.
 - $x^3 - 2x + 5 = 0$
 - $\log_2(x^2 - 9) = 4$.
- Solve $5x + 2y + 4z = 8$, $-3x + y + 2z = -7$, $2x + y + z = 3$ for x, y and z.

Solutions. 1. a) Enter `solve('x^3-2*x+5=0', 'x')`. The only real answer is **-2.09**.
 b) Enter `solve('log(x^2-9)/log(2)=4','x')`. The answers are **ans= 5, -5**.
 2. `[x,y,z]=solve('5*x+2*y+4*z = 8', '-3*x+y+2*z = -7', '2*x+y+z = 3')` **x=2 y=-1 z=0**

3. Representing functions

The following table gives an overview of how most commonly used functions or expressions are represented in Matlab.

To represent a function, use the command **inline**. Similarly to solve, this command is followed by parenthesis and has the following form:

`inline('function', 'independent variable of the function')`

Here is how to define the function $x^2 + 3x - 2$:

`>> f = inline('x^2+3*x-2', 'x')` **f = Inline**
function: f(x) = x^2+3*x-2

function or symbol	representation in MATLAB
e^x	exp(x)
ln x	log(x)
log x	log(x)/log(10)
log. base a of x	log(x)/log(a)
sin x	sin(x)
cos x	cos(x)
arctan(x)	atan(x)
π	pi

After defining a function, we can evaluate it at a point. For example,

`>> f(2)` **ans = 8**

In some cases, we will need to define function f as a vector. Then we use:

`>> f = inline(vectorize('x^2+3*x-2'), 'x')` **f = Inline function: f(x) = x.^2+3.*x-2**

In this case, we can evaluate a function at more than one point at the same time. For example, to evaluate the above function at 1, 3 and 5 we have:

`>> f([1 3 5])` **ans = 2 16 38**

If a function is short, it might be **faster to evaluate a function at a point simply by typing the value of x directly for x**. For example, to evaluate sin(x) at x=2, simply type

`>> sin(2)` and obtain the answer **ans = .909297**

As when using the calculator, one must be careful when representing a function. For example

- $\frac{1}{x(x+6)}$ should be represented as **1/(x*(x+6))** not as **1/x*(x+6)** nor as **1/x(x+6)**,
- $\frac{3}{x^2+5x+6}$ should be represented as **3/(x^2+5*x+6)** not as **3/x^2+5*x+6**,
- e^{5x^2} should be represented as **exp(5*x^2)** not as **e^(5*x^2)**, **exp*(5*x^2)**,

- $\exp(5x^2)$ nor as $\exp^*(5*x^2)$.
- $\ln(x)$ should be represented as $\log(x)$, not $\ln(x)$.
- $\log_3(x^2)$ should be represented as $\log(x^2)/\log(3)$ not as $\log(x)/\log(3)*x^2$.

4. Graphics

Let us start by declaring that x is a variable:

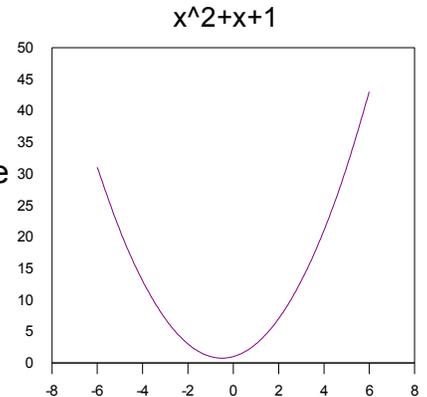
```
>> syms x
```

The simplest command in Matlab for graphing is **ezplot**. The command has the following form

ezplot(function)

For example, to graph the function x^2+x+1 , you simply type

```
>> ezplot(x^2+x+1)
```

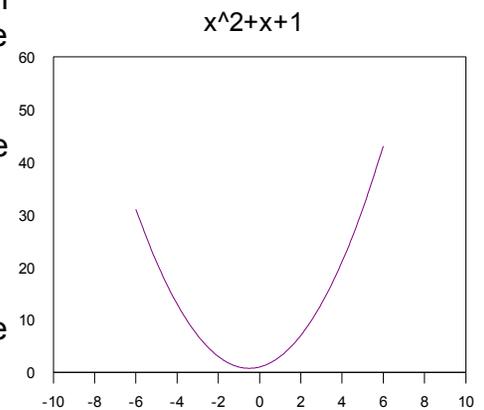


A new window will open and graph will be displayed. To copy the figure to a text file, go to **Edit** and choose **Copy Figure**. Then place cursor to the place in the word file where you want the figure to be pasted and choose **Edit** and **Paste**.

We can specify the different scale on x and y axis. To do this, the command **axis** is used. It has the following form

axis([x_{min} , x_{max} , y_{min} , y_{max}])

This command parallels the commands in menu WINDOW on the TI83 calculators.



For example, to see the above graph between x -values -10 and 10 and y -values 0 and 60 , you can enter

```
>> axis([-10 10 0 60])
```

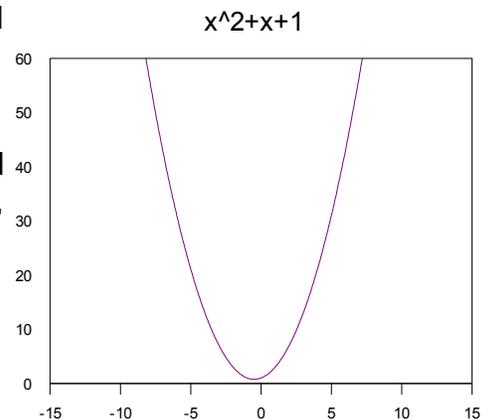
Note that the domain of function did not change by command **axis**. To see the graph on the entire domain (in this case $[-10, 10]$), add that domain after the function in the command **ezplot**:

ezplot(function, [x_{min} , x_{max}])

In the example with x^2+x+1

```
>> ezplot(x^2+x+1, [-10, 10])
```

will give you the desired graph.



For the alternative command for graphics, **plot**, you can find more details by typing **help**.

To plot multiple curves on the same window, you can also use the **ezplot** command. For

example:

```
>> ezplot(sin(x))
>> hold on
>> ezplot(exp(-x^2))
>> hold off
```

5. Parametric Plots

We can use the command **ezplot** to graph a parametric curve as well. For example, to graph a circle $x = \cos t$, $y = \sin t$ for $0 \leq t \leq 2\pi$, we have:

```
>> ezplot('cos(t)', 'sin(t)', [0, 2*pi])
```

Practice problems 3

1. Let $f(x) = \frac{x^3 + x + 1}{x}$

- Represent $f(x)$ as a function in Matlab. Then evaluate it at $x=3$ and $x=-2$.
- Find x -value(s) that corresponds to y -value $y=2$.
- Graph $f(x)$ on domain $[-4, 4]$.

2. Graph $\ln(x+1)$ and $1-x^2$ on the same plot for x in $[-2, 6]$ and y in $[-4, 4]$.

Solutions

1. a) `>> f=inline('(x^3+x+1)/x','x');` `>> f(3)` **ans= 10.333,** `>>f(-2)` **ans=4.5.**

b) The problem is asking you to solve equation $\frac{x^3+x+1}{x}=2$. Using solve command, `solve('(x^3+x+1)/x=2','x')`. you get one real answer $x=-1.3247$

c) `ezplot((x^3+x+1)/x, [-4,4])`.

6. Solving Equations in Matlab using **fzero**

In some cases, the command **solve** may fail to produce all the solutions of an equation. In those cases, you can try to find solutions using **fzero** (short for "find zero") command. In order to use the command, first you need to write equation in the form

$$f(x)=0.$$

Thus, put all the terms of the equations on one side leaving just zero on the other. To find a solution near the x -value $x=a$, you can use

fzero('left side of the equation', a)

The command **fzero**, similarly as **solve** is always followed by parenthesis. The equation should be in single quotes.

If it is not clear what a convenient x -value a should be, you may want to graph the function on the left side of the equation first, check where it intersects the x -axis (alternatively graph left and right side of the equation if it is not in $f(x)=0$ form and see where the two functions intersect) and then decide which solution you need to find.

For example, to solve the equation $e^{x^2}-2=x+4$, we can first graph the functions on the left and right side of the equation using

```
syms x      ezplot(exp(x^2)-2)      hold on      ezplot(x+4)      hold off
```

From the graph, we can see that the two functions intersect at a number near -1 and at a number near 1. To use **fzero**, we need to represent the equation in the form $e^{x^2}-2-(x+4)=0$ (or simplified form $e^{x^2}-x-6=0$). Then, we can find the positive solution by using **fzero** to find a zero near 1 and then to find the negative solution near -1, for example. Thus, both solutions can be obtained by:

```
>> fzero('exp(x^2)-2-(x+4)', -1)      ans = -1.248
>> fzero('exp(x^2)-2-(x+4)', 1)      ans = 1.415
```

Note also that the command **solve('exp(x^2)-2=x+4', 'x')** returns just the positive solution. Thus, knowing how to use **fzero** command may be really useful in some cases.

7. Differentiation

Start by declaring x for a variable. The command for differentiation is **diff**. It has the following form

```
diff(function)
```

For example,

```
>> syms x
>> diff(x^3-2*x+5)
gives us the answer      ans = 3*x^2-2
```

To get n -th derivative use

```
diff(function, n)
```

For example, the 23rd derivative of $\sin(x)$ is obtained as follows.

```
>> diff(sin(x), 23)      ans = -cos(x)
```

Practice problems 4

- Let $f(x) = \frac{x^3+x+1}{x}$ a) Use Matlab to find the first derivative of $f(x)$. b) Evaluate the first derivative at $x=1$.
- Find the 12th derivative of the function $(\frac{x}{2}+1)^{65}$.

Solutions.

- a) `syms x diff((x^3+x+1)/x) ans = 2*x-1/x^2 or (2*x^3-1)/x^2.`

- b) Inline the derivative: `g=inline('2*x-1/x^2','x')`. Then `g(1)` gives you `ans=1`.
 2. `diff((x/2+1)^65, 12)`

8. Integration

We can use Matlab for computing both definite and indefinite integrals using the command `int`. For the indefinite integrals, start with `syms x` followed by the command

`int(function)`

For example, the command

`>> int(x^2)`

evaluates the integral $\int x^2 dx$ and gives us the answer
`ans = 1/3*x^3`

For definitive integrals, the command is

`int(function, lower bound, upper bound)`

For example,

`>> int(x^2, 0, 1)`

evaluates the integral $\int_0^1 x^2 dx$ The answer is `ans = 1/3`

In Matlab, `Inf` stands for positive infinity. Matlab can evaluate the (convergent) improper integrals as well. For example:

`>> int(1/x^2, 1, Inf)` `ans = 1`

For the divergent integrals, Matlab gives us the answer infinity. For example:

`>> int(1/x, 0, 1)` `ans = inf`

Practice problems 5

1. Evaluate the following integrals. a) $\int x e^{-3x} dx$ b) $\int_0^1 x e^{-3x} dx$.
 2. Using Matlab, determine if the following integrals converge or diverge. If they converge, evaluate them.

a) $\int_1^{\infty} \frac{3}{x^2+5x+6} dx$

b) $\int_1^2 \frac{1}{(x-1)(x+1)} dx$

Solutions.

1. a) `syms x` `int(x*exp(-3*x))` `ans=-1/3*x*exp(-3*x)-1/9*exp(-3*x)`
 b) `int(x*exp(-3*x), 0,1)` `ans=-4/9*exp(-3)+1/9` `vpa(ans, 4)` `ans=.08898`
 2. a) `int(3/(x^2+5*x+6), 1, Inf)` `vpa(ans, 4)` `ans= .8338` So, the integral is convergent.
 b) `int(1/((x-1)*(x+1)), 1, 2)` `ans= Inf` So, the integral is divergent.

9. Limits

You can use **limit** to compute limits, left and right limits as well as infinite limits. For example, to evaluate the limit when $x \rightarrow 2$ of the function $\frac{x^2-4}{x-2}$, we have:

```
>> syms x
>> limit((x^2-4)/(x-2), x, 2)      ans = 4
```

For left or right limits:

```
>> limit(abs(x)/x, x, 0, 'left')   ans = -1
>> limit(abs(x)/x, x, 0, 'right')  ans = 1
```

Limits at infinity:

```
>> limit(exp(-x^2-5)+3, x, Inf)    ans = 3
```

Practice problem 6

Find the limits of the following functions at indicated values.

a) $f(x) = \frac{x^{12}-1}{x^3-1}$, $x \rightarrow 1$ b) $f(x) = 3+e^{-2x}$, $x \rightarrow \infty$ c) $f(x) = \frac{6x^3-4x+5}{2x^3-1}$, $x \rightarrow \infty$

Solutions.

```
a) syms x   limit((x^12-1)/(x^3-1), x, 1)      ans=4
b) limit(3+exp(-2*x), x, Inf)                  ans=3
c) limit((6*x^3-4*x+5)/(2*x^3-1), x, Inf)      ans=3
```