Math Methods 1
Lia Vas

Formulas for Exam 1

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \|x\|$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Vectors.** If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$, then
   (a) The **length** of $\vec{a}$ is $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.
   (b) The **dot product** is $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.
   (c) The **cross product** is $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

4. **Line Integrals.** The **work** done by the force $\mathbf{f} = (P, Q, R)$ in moving the particle along the curve C is the line integral

$$W = \int_C \mathbf{f} \cdot d\mathbf{r} = \int_C P dx + Q dy + R dz.$$

5. **Coordinate Substitutions.**
   (a) **Cylindrical Coordinates.**

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z.$$

   Then $x^2 + y^2 = r^2$ and the Jacobian is $J = r$.

   (b) **Spherical Coordinates.**

$$x = r\cos\theta\sin\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\phi.$$

   Then $x^2 + y^2 + z^2 = r^2$ and the Jacobian J is $r^2\sin\phi$.

6. **Parametric Surfaces. Surface and Flux Integrals.**
   Parametric surface is $\mathbf{r}(u, v) = (x(u, v), y(u, v), z(u, v))$.

(a) The tangent plane has the **normal vector** $\mathbf{r}_u \times \mathbf{r}_v = (x_u, y_u, z_u) \times (x_v, y_v, z_v)$.

$$dS = |\mathbf{r}_u \times \mathbf{r}_v|\, dudv$$

$$d\mathbf{S} = \pm(\mathbf{r}_u \times \mathbf{r}_v)\, dudv$$

(b) The **surface integral** of $f(x, y, z)$ is

$$\boxed{\int\int_S f(x,y,z)\; dS = \int\int_S f(\mathbf{r}(u,v))\; |\mathbf{r}_u \times \mathbf{r}_v|\; dudv}$$

If $f = 1$, the integral computes the surface area of the surface $\mathbf{r}(u, v)$

$$S = \int\int_S dS.$$

If $\rho(x, y, z)$ is the density function of a thin sheet S, then the mass m is given by

$$m = \int\int_S \rho(x,y,z)\; dS$$

(c) The **flux integral** of $\mathbf{f} = (P, Q, R)$ is

$$\boxed{\int\int_S \mathbf{f} \cdot d\mathbf{S} = \int\int_S \mathbf{f} \cdot \mathbf{n}\; dS = \pm\int\int_S \mathbf{f} \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dudv}$$

If $\mathbf{f}$ is a force field, then this integral computes the flux of the field – the total effect of the field acting over the region S.

7. **Gradient, Curl, and Divergence.**

The gradient operator is $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$. If $\mathbf{f} = (P, Q, R)$ is a vector function then the **divergence** is

$$\text{div}\; \mathbf{f} = \nabla \cdot \mathbf{f} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

and the **curl** is

$$\text{curl}\; \mathbf{f} = \nabla \times \mathbf{f} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k}$$

8. **Stokes' Theorem.**

$$\boxed{\oint_C \mathbf{f} \cdot d\mathbf{r} = \int\int_S \text{curl}\mathbf{f} \cdot d\mathbf{S}}$$

9. **Divergence Theorem.**

$$\boxed{\int\int_S \mathbf{f} \cdot d\mathbf{S} = \int\int\int_V \text{div}\mathbf{f}\;\; dx\; dy\; dz}$$