

## Formulas for Exam 2

### 1. Derivatives.

$y$	$x^n$	$e^x$	$b^x$	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
$y'$	$nx^{n-1}$	$e^x$	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

### 2. Integrals.

$y$	$x^n$	$e^x$	$b^x$	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y dx$	$\frac{1}{n+1}x^{n+1}$	$e^x$	$\frac{1}{\ln b} b^x$	$\ln  x $	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

### 3. Complex numbers.

$$z = x + iy = r \cos \theta + ir \sin \theta = re^{i\theta}.$$

where  $r = |z| = \sqrt{x^2 + y^2}$ .

**Euler's formula.**  $\cos t + i \sin t = e^{it}$ .

### 4. Analytic functions. Let $f(z) = f(x + iy) = u(x, y) + iv(x, y)$ .

**Cauchy-Riemann equations.**

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

**Laplace Equations.**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

### 5. Cauchy's Theorem. If $f(z)$ is analytic and $C$ is a closed curve, then

$$\oint_C f(z) dz = 0.$$

**Cauchy's Integral Formula.**  $f^{(n)}(a) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}} dz$

### 6. Laurent Series. If $f$ is analytic at $z = a$ then $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n$ where

$$a_n = \frac{f^{(n)}(a)}{n!} = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}} dz.$$

If  $z = a$  is an isolated singularity of  $f$  then

$$f(z) = \sum_{n=-\infty}^{\infty} a_n(z-a)^n \quad \text{where} \quad a_n = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}} dz$$

The coefficient  $a_{-1}$  of the term with  $\frac{1}{z-a}$  in the expansion for  $f(z)$  is the **residue** of  $f(z)$  at  $a$ .

The residue at a **pole**  $z = a$  of **order**  $n$  is

$$a_{-1} = \frac{1}{(n-1)!} \lim_{z \rightarrow a} \frac{d^{n-1}}{dz^{n-1}} ((z-a)^n f(z)).$$

**Some elementary function expansions.**

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} \quad \frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \quad \sin z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!} \quad \cos z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$$

**The Residue Theorem.** If  $z = a$  is an isolated singularity of  $f(z)$  and  $C$  is a closed, piecewise smooth, positive oriented curve whose interior contains  $a$ , then

$$\oint_C f(z) dz = 2\pi i a_{-1} = 2\pi i (\text{coefficient of the term with } \frac{1}{z-a}).$$

If the interior of the curve  $C$  contains the isolated singularities  $z_1, z_2, \dots, z_n$  of  $f$  and  $R_1, R_2, \dots, R_n$  are the residues at  $z_1, z_2, \dots, z_n$ , then

$$\oint_C f(z) dz = 2\pi i (R_1 + R_2 + \dots + R_n).$$