Math Methods 1
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Formulas for Exam 2

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\ dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b}\ b^x$	$\ln \lvert x\rvert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Cylindrical coordinates.** $\mathbf{r} = (r\cos\theta, r\sin\theta, z) \Rightarrow$

$$\hat{\mathbf{r}}_r = \mathbf{r}_r = (\cos\theta, \sin\theta, 0),\ \hat{\mathbf{r}}_\theta = \tfrac{1}{r}\mathbf{r}_\theta = (-\sin\theta, \cos\theta, 0), \text{ and } \hat{\mathbf{r}}_z = \mathbf{r}_z = (0,0,1).$$

Conversion equations: $\hat{\mathbf{r}}_r = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$, $\quad \hat{\mathbf{r}}_\theta = -\sin\theta\mathbf{i} + \cos\theta\mathbf{j}$, $\quad$ and $\quad \hat{\mathbf{r}}_z = \mathbf{k}$.
$\mathbf{i} = \cos\theta\hat{\mathbf{r}}_r - \sin\theta\hat{\mathbf{r}}_\theta$, $\quad \mathbf{j} = \sin\theta\hat{\mathbf{r}}_r + \cos\theta\hat{\mathbf{r}}_\theta$, $\quad$ and $\quad \mathbf{k} = \hat{\mathbf{r}}_z$.

$$d\mathbf{r} = \hat{\mathbf{r}}_r dr + r\hat{\mathbf{r}}_\theta d\theta + \mathbf{r}_z dz.$$

$$dV = r\ dr\, d\theta\, dz$$

If S is a vertical cylinder of radius r centered at the origin, $d\mathbf{S} = r d\theta dz \hat{\mathbf{r}}_r$.

If $\mathbf{f} = P\hat{\mathbf{r}}_r + Q\hat{\mathbf{r}}_\theta + R\hat{\mathbf{r}}_z$, then

$$\text{div}\mathbf{f} = \nabla\cdot\mathbf{f} = \frac{1}{r}\frac{\partial(rP)}{\partial r} + \frac{1}{r}\frac{\partial Q}{\partial\theta} + \frac{\partial R}{\partial z}$$

$$\text{curl}\mathbf{f} = \nabla\times\mathbf{f} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}}_r & r\hat{\mathbf{r}}_\theta & \hat{\mathbf{r}}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ P & rQ & R \end{vmatrix}$$

4. **Spherical coordinates.** $\mathbf{r} = (r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi) \Rightarrow$

$$\hat{\mathbf{r}}_r = \mathbf{r}_r = (\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi),\ \hat{\mathbf{r}}_\phi = \tfrac{1}{r}\mathbf{r}_\phi = (\cos\theta\cos\phi, \sin\theta\cos\phi, -\sin\phi), \text{ and}$$
$$\hat{\mathbf{r}}_\theta = \tfrac{1}{r\sin\phi}\mathbf{r}_\theta = (-\sin\theta, \cos\theta, 0).$$

Conversion equations: $\hat{\mathbf{r}}_r = \cos\theta\sin\phi\mathbf{i} + \sin\theta\sin\phi\mathbf{j} + \cos\phi\mathbf{k}$, $\hat{\mathbf{r}}_\phi = \cos\theta\cos\phi\mathbf{i} + \sin\theta\cos\phi\mathbf{j} - \sin\phi\mathbf{k}$, and $\hat{\mathbf{r}}_\theta = -\sin\theta\mathbf{i} + \cos\theta\mathbf{j}$.

$$d\mathbf{r} = \hat{\mathbf{r}}_r dr + r\mathbf{r}_\phi d\phi + r\sin\phi\hat{\mathbf{r}}_\theta d\theta.$$

$$dV = dxdydz = r^2\sin\phi\ dr\, d\theta\, d\phi$$

If S is a sphere of radius r, then $d\mathbf{S} = r^2 \sin\phi \, d\phi \, d\theta \, \hat{\mathbf{r}}_r$.

If $\mathbf{f} = P\hat{\mathbf{r}}_r + Q\hat{\mathbf{r}}_\phi + R\hat{\mathbf{r}}_\theta$, then

$$\mathrm{div}\mathbf{f} = \nabla \cdot \mathbf{f} = \frac{1}{r^2}\frac{\partial(r^2 P)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial(\sin\phi \, Q)}{\partial\phi} + \frac{1}{r\sin\phi}\frac{\partial R}{\partial\theta}$$

$$\mathrm{curl}\mathbf{f} = \nabla \times \mathbf{f} = \frac{1}{r^2\sin\phi}\begin{vmatrix} \hat{\mathbf{r}}_r & r\hat{\mathbf{r}}_\phi & r\sin\phi\hat{\mathbf{r}}_\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial\theta} \\ P & rQ & r\sin\phi \, R \end{vmatrix}$$

5. **Stokes' and Divergence Theorems.**

$$\oint_C \mathbf{f} \cdot d\mathbf{r} = \int\int_S \mathrm{curl}\mathbf{f} \cdot d\mathbf{S}$$

$$\int\int_S \mathbf{f} \cdot d\mathbf{S} = \int\int\int_V \mathrm{div}\mathbf{f} \; dV$$

6. **Complex numbers.**

$$z = x + iy = r\cos\theta + ir\sin\theta = re^{i\theta}.$$

where $r = |z| = \sqrt{x^2 + y^2}$.

Euler's formula.

$$\cos t + i\sin t = e^{it}.$$

7. **Analytic functions.** Let $f(z) = f(x + iy) = u(x, y) + iv(x, y)$.

Cauchy-Riemann equations.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Laplace Equations.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

8. **Cauchy's Theorem.** If $f(z)$ is analytic and C is a closed curve, then $\oint_C f(z)dz = 0$.

Cauchy's Integral Formula. $f^{(n)}(a) = \frac{n!}{2\pi i}\oint_C \frac{f(z)}{(z-a)^{n+1}} \, dz$

9. **Laurent Series.** If f is analytic at $z = a$ then $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n$ where $a_n = \frac{f^{(n)}(a)}{n!} = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)^{n+1}} \, dz$. If $z = a$ is an isolated singularity of f then

$$f(z) = \sum_{n=-\infty}^{\infty} a_n(z-a)^n \quad \text{where} \quad a_n = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)^{n+1}} \, dz$$

The coefficient a_{-1} is the residue of f at a. **Some elementary function expansions.**

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} \qquad \frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \qquad \sin z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!} \qquad \cos z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$$