

Formulas for Exam 3

1. Derivatives.

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. Integrals.

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b} b^x$	$\ln x $	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. Integration by parts $\int u dv = uv - \int v du$

4. Fourier Series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi nx}{T} + b_n \sin \frac{2\pi nx}{T} \right)$$

$$a_n = \frac{2}{T} \int_{x_0}^{x_0+T} f(x) \cos \frac{2n\pi x}{T} dx \qquad b_n = \frac{2}{T} \int_{x_0}^{x_0+T} f(x) \sin \frac{2n\pi x}{T} dx$$

If the interval $(x_0, x_0 + T)$ is of the form $(-L, L)$ (thus $T = 2L$), the following holds.

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx \qquad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

5. Symmetry considerations. If $f(x)$ is even and defined on $(-L, L)$, then

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx, \quad b_n = 0, \quad \text{and} \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}.$$

If $f(x)$ is odd and defined on $(-L, L)$, then

$$a_n = 0, \quad b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx, \quad \text{and} \quad f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}.$$

The above formulas are also used for Fourier cosine and Fourier sine expansions respectively.

6. **Complex Fourier Series.**

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{2n\pi ix}{T}} = \sum_{n=-\infty}^{\infty} c_n \left(\cos \frac{2n\pi x}{T} + i \sin \frac{2n\pi x}{T} \right) \text{ where } c_n = \frac{1}{T} \int_{x_0}^{x_0+T} f(x) e^{-\frac{2n\pi ix}{T}} dx.$$

If $f(x)$ is a real function, $a_n = c_n + c_{-n}$ and $b_n = i(c_n - c_{-n})$ for $n > 0$ and $a_0 = 2c_0$.

7. **Parseval's Theorem.**

$$\frac{1}{T} \int_{x_0}^{x_0+T} |f(x)|^2 dx = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

Parseval's Theorem with Symmetry Considerations. If $f(x)$ is either even or odd function defined on $(-L, L)$,

$$\frac{1}{L} \int_0^L |f(x)|^2 dx = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

8. **Fourier Transform.**

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Inverse Fourier transform.

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

Symmetry considerations.

If $f(t)$ is even, $F(\omega) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \cos \omega t dt$. If $F(\omega)$ is even, $f(t) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F(\omega) \cos \omega t d\omega$.

If $f(t)$ odd, $F(\omega) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \sin \omega t dt$. If $F(\omega)$ is odd, $f(t) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F(\omega) \sin \omega t d\omega$.

9. **Sinc function.**

$$\text{sinc } x = \frac{\sin x}{x}$$

10. **Euler's formula**, related formulas, and symmetry of sine and cosine.

$$e^{ix} = \cos x + i \sin x$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\cos(-x) = \cos x$$

$$\sin(-x) = -\sin x$$