Math Methods 1
Lia Vas

Formulas for Exam 3

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1} x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \lvert x \rvert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Integration by parts** $\int u\, dv = u\, v - \int v\, du$

4. **Complex integrals and Laurent Series.** If $z = a$ is an isolated singularity of f then

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^n \quad \text{where} \quad a_n = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}}\, dz$$

The coefficient a_{-1} is the residue of f at a. **Some elementary function expansions.**

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} \qquad \frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \qquad \sin z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!} \qquad \cos z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$$

The Residue Theorem. If $z = a$ is an isolated singularity of $f(z)$ and C is a closed, piecewise smooth, positive oriented curve whose interior contains a, then

$$\oint_C f(z)\, dz = 2\pi i a_{-1} = 2\pi i (\text{ coefficient of the term with} \frac{1}{z-a}).$$

If the interior of the curve C contains the isolated singularities $z_1, z_2, \ldots z_n$ of f and $R_1, R_2, \ldots R_n$ are the residues at $z_1, z_2, \ldots z_n$, then

$$\oint_C f(z)\, dz = 2\pi i (R_1 + R_2 + \ldots + R_n).$$

The residue at a pole $z = a$ of order n.

$$a_{-1} = \frac{1}{(n-1)!} \lim_{z \to a} \frac{d^{n-1}}{dz^{n-1}} \left((z-a)^n f(z) \right).$$

5. **Fourier Series**.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{2\pi n x}{T} + b_n \sin \frac{2\pi n x}{T})$$

$$a_n = \frac{2}{T} \int_{x_0}^{x_0+T} f(x) \cos \frac{2n\pi x}{T}\, dx \qquad b_n = \frac{2}{T} \int_{x_0}^{x_0+T} f(x) \sin \frac{2n\pi x}{T}\, dx$$

If the interval (x_0, x_0+T) is of the form $(-L, L)$ (thus $T = 2L$), the following holds.

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx \qquad\qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L})$$

6. **Symmetry considerations.** If $f(x)$ is even and defined on $(-L, L)$, then

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx, \quad b_n = 0, \text{ and } \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L}.$$

If $f(x)$ is odd and defined on $(-L, L)$, then

$$a_n = 0, \quad b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx, \quad \text{and} \quad f(x) = \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}.$$

7. **Complex Fourier Series**.

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{2n\pi i x}{T}} = \sum_{n=-\infty}^{\infty} c_n\left(\cos\frac{2n\pi x}{T} + i\sin\frac{2n\pi x}{T}\right) \text{ where } c_n = \frac{1}{T}\int_{x_0}^{x_0+T} f(x)e^{\frac{-2n\pi i x}{T}}\,dx.$$

If $f(x)$ is a real function, $c_n = \frac{1}{2}(a_n - ib_n)$ and $c_{-n} = \frac{1}{2}(a_n + ib_n)$ for $n > 0$.

Parseval's Theorem.

$$\frac{1}{T}\int_{x_0}^{x_0+T} |f(x)|^2 dx = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2).$$

Parseval's Theorem with Symmetry Considerations. If $f(x)$ is either even or odd function defined on $(-L, L)$,

$$\frac{1}{L}\int_0^L |f(x)|^2 dx = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2).$$

8. **Fourier Transform.**

$$F(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

Inverse Fourier transform.

$$f(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega$$

Symmetry considerations.

If $f(t)$ is even, $F(\omega) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(t)\cos\omega t\;dt$. If $F(\omega)$ is even, $f(t) = \sqrt{\frac{2}{\pi}}\int_0^\infty F(\omega)\cos\omega t\;d\omega$.

If $f(t)$ odd, $F(\omega) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(t)\sin\omega t\;dt$. If $F(\omega)$ is odd, $f(t) = \sqrt{\frac{2}{\pi}}\int_0^\infty F(\omega)\sin\omega t\;d\omega$.