Math Methods 1
Lia Vas

Formulas for Exam 4

1. **Derivatives.**

y	x^n	e^x	b^x	$\ln x$	$\log_b x$	$\sin x$	$\cos x$	$\sin^{-1} x$	$\tan^{-1} x$	$\sec^{-1} x$
y'	nx^{n-1}	e^x	$b^x \ln b$	$\frac{1}{x}$	$\frac{1}{x} \cdot \frac{1}{\ln b}$	$\cos x$	$-\sin x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$

2. **Integrals.**

y	x^n	e^x	b^x	$\frac{1}{x}$	$\sin x$	$\cos x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$
$\int y\, dx$	$\frac{1}{n+1}x^{n+1}$	e^x	$\frac{1}{\ln b}\, b^x$	$\ln \lvert x \rvert$	$-\cos x$	$\sin x$	$\sin^{-1} x$	$\tan^{-1} x$

3. **Series Solutions. Regular Point.** If $x = x_0$ is a **regular point** of $y'' + p(x)y' + q(x)y = 0$ a solution can be represented as

$$y = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

The general solutions is $y = c_1 y_1 + c_2 y_2$.

When finding the closed form of the solutions, the following may be useful.

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!}\, x^n \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\, x^{2n+1} \qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\, x^{2n}$$

The series for $e^x, \sin x$ and $\cos x$ converge for every value of x and the series for $\frac{1}{1-x}$ converges on $(-1, 1)$.

4. **Groups.** Group axioms. A group is a nonempty set G with operation $\cdot$ such that

   A1. If a, b are elements of G, then $a \cdot b$ is also an element of G, i.e. the operation is **closed**.

   A2. The operation is **associative**: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for any a, b, c in G.

   A3. There is an **identity element** 1 so that $a \cdot 1 = 1 \cdot a = a$ for every element a of G.

   A4. Every element a has the **inverse** a^{-1}, i.e. $a \cdot a^{-1} = a^{-1} \cdot a = 1$.

5. **Classes of groups and their presentations.**

   (a) If we know the presentations of two groups G_1 and G_2, the presentation of $G_1 \times G_2$ is obtained using the following.

   - The generators of $G_1 \times G_2$ are the generators G_1 and the generators of G_2.
   - The relations on $G_1 \times G_2$ are those of G_1, those of G_2, plus the relations that assert that the generators of G_1 commute with the generators of G_2.

(b)

Group	notation	no. of el.	presentation
Cyclic of order n	C_n	n	$\langle a \mid a^n = 1 \rangle$
Product of two cyclic	$C_n \times C_m$	mn	$\langle a, b \mid a^n = b^m = 1, ba = ab \rangle$
Dihedral	D_n	$2n$	$\langle a, b \mid a^n = b^2 = 1, ba = a^{n-1}b \rangle$
Product of D_n and C_m	$D_n \times C_m$	$2nm$	$\langle a, b, c \mid a^n = b^2 = c^m = 1, ba = a^{n-1}b, ca = ac, bc = cb \rangle$

(c) Direct product of finite cyclic groups C_m and C_n:

$$\boxed{C_m \times C_n \text{ is isomorphic to } C_{mn} \text{ if and only if } m \text{ and } n \text{ are relatively prime}}$$

Relatively prime = no common factor larger than 1.

6. **Point groups.**

Non-linear molecules.

- If there is just one generator, the group is cyclic C_n (C_n, C_{2n}, C_s or C_i for chemists).
- If there are two generators, a rotation a of order n and a reflection b, the group is either $C_n \times C_2$ or D_n.

  If b is the symmetry with respect to the horizontal plane, then a and b commute. In this case, the group is $C_n \times C_2 = \langle a, b \mid a^n = 1, b^2 = 1, ba = ab \rangle$ (C_{nh} for chemists).

  If b is the reflection with respect to a vertical plane, then a and b do not commute and $ba = a^{-1}b$. In this case, the group is $D_n = \langle a, b \mid a^n = 1, b^2 = 1, ba = a^{n-1}b \rangle$ (C_{nv}, D_n or D_{nd} for chemists).
- If there are three generators: rotation a, symmetry with respect to a vertical plane b, and a symmetry with respect to the horizontal plane c, then $D_n \times C_2 = \langle a, b, c \mid a^n = b^2 = c^2 = 1, ba = a^{n-1}b, bc = cb, ac = ca \rangle$ (D_{nh} for chemists).

Linear molecules.

- If there is a reflection b with respect to the horizontal plane, no reflection with respect to a vertical plane and if x is any rotation in C_∞, then $bx = xb$. Then, the group is $C_\infty \times C_2$ ($C_{\infty h}$ for chemists).
- If there is a reflection b with respect to a vertical plane, no reflection with respect to the horizontal plane, and if x is any rotation in C_∞, then $bx = x^{-1}b$. In this case, the groups is D_∞ ($C_{\infty v}$ or D_∞ for chemists).
- If there is a reflection b with respect to a vertical plane, a reflection c with respect to the horizontal plane, and if x is any rotation in C_∞, then the following relations hold $c^2 = b^2 = 1, bx = x^{-1}b, cx = xc$ and the group is $D_\infty \times C_2$ ($D_{\infty h}$ for chemists).

Chemistry and Mathematics notation.

Chem.	Math.	no. of el.	presentation
C_n	C_n	n	$\langle a \mid a^n = 1 \rangle$
C_{nh}	$C_n \times C_2$	$2n$	$\langle a, b \mid a^n = b^2 = 1, ba = ab \rangle$
C_{nv}	D_n	$2n$	$\langle a, b \mid a^n = b^2 = 1, ba = a^{n-1}b \rangle$
C_i, C_s	C_2	2	$\langle b \mid b^2 = 1 \rangle$
D_n	D_n	$2n$	$\langle a, b \mid a^n = b^2 = 1, ba = a^{n-1}b \rangle$
D_{nh}	$C_{nv} \times C_2 = D_n \times C_2$	$4n$	$\langle a, b, c \mid a^n = b^2 = c^2 = 1, ba = a^{n-1}b, ca = ac, bc = cb \rangle$
D_{nd}	D_{2n}	$4n$	$\langle a, b \mid a^{2n} = 1, b^2 = 1, ba = a^{2n-1}b \rangle$
S_{2n}	C_{2n}	$2n$	$\langle a \mid a^{2n} = 1 \rangle$
I	A_5	60	$\langle a, b \mid a^2 = b^3 = (ab)^5 = 1 \rangle$
I_h	$A_5 \times C_2$	120	$\langle a, b, c \mid a^2 = b^3 = (ab)^5 = 1, ac = ca, bc = cb \rangle$
O	S_4	24	$\langle a, b \mid a^2 = b^3 = (ab)^4 = 1 \rangle$
O_h	$S_4 \times C_2$	48	$\langle a, b, c \mid a^2 = b^3 = (ab)^4 = 1, ac = ca, bc = cb \rangle$
T	A_4	12	$\langle a, b \mid a^2 = b^3 = (ab)^3 = 1 \rangle$
T_h	$A_4 \times C_2$	24	$\langle a, b, c \mid a^2 = b^3 = (ab)^3 = 1, ac = ca, bc = cb \rangle$
T_d	S_4	24	$\langle a, b \mid a^2 = b^3 = (ab)^4 = 1 \rangle$
C_∞	$C_\infty = SO(2, R)$	∞	no finite presentation
$C_{\infty v}$	D_∞	∞	no finite presentation
$C_{\infty h}$	$C_\infty \times C_2$	∞	no finite presentation
D_∞	D_∞	∞	no finite presentation
$D_{\infty h}$	$D_\infty \times C_2$	∞	no finite presentation