Math Methods
Lia Vas

Project Topics and Report Instructions

You can turn in your project report any time during the semester but not later than the end of **the second week of April.**

Project Overview. Students should be able to research a topic or learn about a method on their own as well, not just via an extensive classroom instruction. The purpose of this project is to practice this skill. You can choose a topic from the **sample topics list below**. You can also pick a topic that is not listed below, but should consult with me to double check it is relevant and extensive enough.

The topics below can be divided into two categories:

1. Topics concentrated on applications of mathematics to physics or related sciences. Any of these topics should deepen your understanding of *applications* of certain mathematical method.

2. Topics concentrated on mathematical proof of a concept covered in class. Any of these topics should enhance your ability to follow, understand and adopt arguments involved in *mathematical proofs.*

If you end up researching a topic especially thoroughly and beyond the scope of the published material you have been using, you will be able to present your findings during **the USciences Annual Research Day** or even to consider preparing your findings for a publication. You can consult with me about this. Keep in mind that the **Research Day is usually in mid April and the abstract is due in mid March**.

Report Instructions. When you pick a topic and find some material that you may need to cover your topic, you should write a project report. The purpose of the report is to practice your skills of effectively and convincingly writing about your methods, results and conclusions obtained.

The report should consist of three parts: the introduction, the main part and the conclusion. Throughout the report, make sure that your sentences are clear and your spelling, grammar and punctuation correct. Avoid long sentences and do not use complicated words when you can communicate something using simple and clear phrases. The technical level of your report should be such that your class peers will be able to understand and follow your arguments.

In the **introduction**, you should state the main goal of the project. You can start with the problem statement and the background of the problem. You should clearly state the project objective. Then you can also give a brief summary of the methods used and conclusions obtained.

In the **main part**, you should list your results and conclusions. You should explain the mathematical methods used, prove the main statement using well supported mathematically correct claims, or explain how the mathematical methods apply to physics, chemistry or other sciences. All mathematical data manipulation, calculations, algebraic or numerical work should be in the main part. Make sure that each step is clear and justified.

Finally, you should list your **conclusions** or data testifying the validity and effectiveness of your methods.

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List of Sample Project Topics

I. Surface Integrals. Stokes' and Divergence Theorems.

1. **Green's Theorem**. Prove the theorem and illustrate its use with examples. Related material can be found in section 11.3 of Riley/Hobson/Bence textbook.

2. **Physics applications of Stokes' and Divergence Theorems**. Related material can be found in sections 11.8.3 and 11.9.2 of Riley/Hobson/Bence textbook.

3. **Parametrization of Surfaces of Revolution. Surface Area of a Torus.** This topic is for students who will not take Differential Geometry (MA 430). Relevant material can be found in the MA 430 handout "Surfaces part 2" on my website.

4. **Ruled Surfaces and parametrization of the Möbius strip.** This topic is for students who will not take Differential Geometry (MA 430). Relevant material can be found in the MA 430 handout "Surfaces part 2" on my website.

II. Complex Analysis

5. **Elementary Complex Functions.** Euler formula, complex functions $e^z, \sin z$, $\cos z$ and their inverses. The relevant material can be found in sections 3.3, 3.5, and 24.4 of Riley/Hobson/Bence textbook.

6. **Power Series Expansions of Elementary Complex Functions.** Prove the Uniqueness Theorem (the first half of section 24.11 of Riley/Hobson/Bence textbook). Use this theorem to explain the arguments made in class notes (bottom of page 6, Complex Functions handout).

7. **Cauchy's Differential and Integral formula**. Prove these two formulas. Related material can be found in sections 24.9 and 24.10 of Riley/Hobson/Bence textbook.

8. **Formula that computes the residue at a pole of order k.** Prove the formula for the residue at a pole of order k that involves the limit. Illustrate the use of this formula with examples. The material for this topic can be found in the second half of section 24.11 of Riley/Hobson/Bence textbook.

9. **Residue theorem**. Prove the theorem and illustrate its use with examples. The material for this topic can be found in section 24.12 of Riley/Hobson/Bence textbook.

10. **Physics Application – Complex Potentials**. The material for this topic can be found in section 25.1 of Riley/Hobson/Bence textbook.

11. **Parseval's Theorem**. Prove this theorem and illustrate its use with examples. The material for this topic can be found in section 12.8 of Riley/Hobson/Bence textbook.

III. Fourier Series and Transform

12. **The Fourier Series Coefficients and the formula for Fourier Transform**. Prove the formulas computing the Fourier Series Coefficients and the Fourier Transform. Start with the ideas presented on the class handout on Fourier Series and then use section 12.2 of Riley/Hobson/Bence textbook. Then turn to coefficients of complex Fourier series (section 12.7 of Riley/Hobson/Bence textbook).

13. **Fourier Transform in higher dimensions**. The material for this topic can be found in section 13.1.10 of Riley/Hobson/Bence textbook. Include some examples from this section too.

14. **Computing Fourier Transform in Matlab.** Explain how to find Fourier transform of a function using Matlab. Illustrate your findings with multiple examples.

15. **Physics Application – Heisenberg Uncertainty principle**. The material for this topic can be found in section 13.1.1 of Riley/Hobson/Bence textbook.

16. **Physics Application – Fraunhofer Diffraction**. The material for this topic can be found in section 13.1.2 of Riley/Hobson/Bence textbook.

IV. Series Solutions of Ordinary Differential Equations

17. **Finding the second solution using the derivative method**. The derivative method is used to obtain the form of the second solution in case that the difference of zeros of index equation is an integer. Explain this method and the formula for the second solution. The material for this topic can be found in section 16.4.2 of Riley/Hobson/Bence textbook.

18. **Linear Independence of Solutions**. This topics is for students who took Linear Algebra, MA 316. Explain what is Wronskian and how it is used to determine linear independence of solutions. Illustrate your claims with examples. The material for this topic can be found in section 16.1 (before 16.1.1) and 16.4.1.

19. **Polynomial Solutions.** Sometimes the solutions of second order linear homogeneous differential equation can be found to be polynomials, not infinite series. This topic focuses on such cases. The material for this topic can be found in section 16.5 of Riley/Hobson/Bence textbook. Illustrate your claims with examples.

20. **Methods of solving second-order linear equations with non-constant coefficients**. This topic is for students who took Differential Equations, MA 320. At the beginning of your class handout on series solutions, the overview of three methods of solving higher-order equations with non-constant coefficients, Legendre's (and Euler's), Exact and Partially Known Complementary Function equations, was presented. Expand this overview into more complete account of the three methods and illustrate each method with examples. The material for this topic can be found in sections 15.2.1, 15.2.2, and 15.2.3 of Riley/Hobson/Bence textbook.

21. **Physics Application – Stokes equation**. Material for this topics can be found in sections 25.6 (first two parts) of Riley/Hobson/Bence textbook.

Optional addition: you can also include discussion on WKB methods and phase memory (section 25.7).

V. Group Theory. Symmetry Groups

22. **Group Axioms and Cyclic Groups**. For this project topic, you can (1) Prove that a set G is a group (by definition given in class) if and only if the rules A1, A2 and D hold. The outline of the proof of this claim can be found on page 4 of the class handout on groups. (2) Expand on cyclic groups. The handout on my website (Math Methods, under "Project Topics") for more details.

23. **Groups with small number of elements.** More details can be found on my website (Math Methods, under "Project Topics").

24. **Cycle graphs of groups.** More details can be found on my website (Math Methods, under "Project Topics").

25. **Symmetric and Alternating Groups**. Expand on symmetric groups S_n, give definition and list properties of alternating groups A_n. You can start by going over the material on S_n and A_n in the class notes and then supplement it with the material from the web and section 28.4 of Riley/Hobson/Bence textbook.

26. **Representations of groups.** More details can be found on my website (Math Methods, under "Project Topics"). You can also include an outline of the way how the point group representations are used to make conclusions about the structural properties and energy levels of the molecule. A web search can produce sufficient material.

27. **Math and Chemistry**. This topics is for students who took Inorganic Chemistry, CH 431. We have seen that some of the different point groups are, in fact, isomorphic. So, for mathematicians, these groups do not have any significant differences. However, from a chemist's point of view, they are significantly different. For example, C_i and C_s are different for a chemist, but for a mathematician, both are just C_2. Then point groups C_{nv} and D_n are both the same (mathematical) dihedral group of $2n$ elements.

    Taking in account the differences in geometry of corresponding molecules, explain why such groups are considered different by a chemist. Also, expand on the differences in notation in point groups and mathematics and chemistry. A good places to start are mathworld.wolfram.com. and wikipedia.org.

    Optional addition: There are several online multimedia programs designed to identify symmetry elements and assign point groups to molecules. One of them, the Point Group Tutorial, can be downloaded at http://www.chemistry.emory.edu/pointgrp/ Demonstrate the use of this or similar computer applications.