TORSION THEORIES FOR GROUP VON NEUMANN ALGEBRAS

LIA VAS

ABSTRACT. The use of group Von Neumann algebras facilitates the study of homology of certain spaces. The study of modules over a group von Neumann algebra $\mathcal{N}G$ can be improved by the use of torsion theories. In this work, some torsion theories for $\mathcal{N}G$ are presented, compared and studied. The torsion and torsion-free classes of some of these theories are related to the classes studied by other authors. Using the torsion theories, the class of finitely generated modules over $\mathcal{N}G$ is described in more details. From that description, a useful criterion for checking if a finitely generated $\mathcal{N}G$-module is flat and the formula for computing its capacity are obtained. Also, the result on the isomorphism of K_0 of $\mathcal{N}G$ and its algebra of affiliated operators $\mathcal{U}G$ is improved. Then, the behavior of the torsion and torsion-free classes of the torsion theories of interest under the induction of a module with respect to inclusion of a group von Neumann algebra of a subgroup of G in the algebra $\mathcal{N}G$ is studied. Using these results, the formula for the capacity of the induced module is improved.

The torsion theories for the algebra $\mathcal{U}G$ are studied as well. It is shown that they have the same properties as their analogues for $\mathcal{N}G$ plus some additional properties. These additional properties result from the ring-theoretic features of $\mathcal{U}G$ that $\mathcal{N}G$ does not necessarily have.

If certain torsion theories, different in general, are equal for a particular $\mathcal{N}G$, then such $\mathcal{N}G$ and $\mathcal{U}G$ have some additional ring-theoretic properties. In particular, the necessary and sufficient conditions for $\mathcal{U}G$ to be semisimple are studied.

In the case of $\mathcal{U}G$ not being semisimple, the left and right global dimension of $\mathcal{U}G$ are calculated and an upper bound for the left and right global dimension of $\mathcal{N}G$ given. These results are proven under the assumption of the Continuum Hypothesis.

A group von Neumann algebra is just one example of a finite von Neumann algebra. Most of the results proven here for a group von Neumann algebra hold for any finite von Neumann algebra without any modification. A few of the results have to be modified slightly before stated and proven in this greater generality.